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ALGORITHMS FOR ASSESSING THE QUALITATIVE AND QUANTITATIVE RISKS OF LENDING TO SMALL AND MEDIUM-SIZED BUSINESSES BASED ON FUZZY CALCULUS

Sergey Sveshnikov*, Victor Bocharnikov, Tatjana Uvarova and Petr Kovalchuk

Consulting company "INEX-FT", Puluja str., 3/270, Kiev, Ukraine

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Abstract

Today, crisis conditions in the economy and finance require high-quality risk assessment. In the article, the authors propose two algorithms for assessing the projects lending risks (PLRs) to small and medium-sized businesses. To assess qualitative PLRs, we proposed to use a hierarchical system of criteria, in which the importance of the criteria is described using the Sugeno fuzzy measure, and the generalized estimate of the qualitative risk is calculated using the Sugeno fuzzy integral. To evaluate quantitative PLRs, we proposed to use the characteristics of fuzzy numbers that describe the project effectiveness criteria and have an arbitrary-form membership function. In addition, to describe quantitative risks, we proposed to use the risk-function of a fuzzy number, which reflects not only the size of possible losses, but also the possibility of their occurrence. This allows you to comprehensively and objectively assess the level of risks. We have demonstrated and discussed this algorithms on the example of preparing data for making a decision on lending to a project for the production of corn syrup in Ukraine.

Keywords: risk, assessing, fuzzy measure, fuzzy integral, fuzzy number

1. INTRODUCTION

The main interest of credit organizations when lending to any small-scale project is the confidence that the borrower will repay the loan body and the loan interest. There are usually two problems here. First, it is necessary to assess the risks of the project in absolute terms, based on the structure and content of the project. And secondly, if the project meets the established criteria, assess its impact on the portfolio of projects of the credit organization, considering alternative investments. In this article, we consider the

* *Corresponding author: sv367@ukr.net*

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solution of the first problem.

To assess risks, managers of a credit organizations must delve into the project, understand the nature and content of the project, and the critical points of the project. Small and medium business projects are associated with a large number of heterogeneous risks which can be caused by climate change, natural disasters, negative changes in pricing, changes in industry and local conditions and other factors. In small and medium business businessmen often do not have sufficient savings and turn to financial organizations to attract credit resources.

A large number of small projects require the lender's management to use a single unified model (template) for risk analysis. This provides a reduction in labor costs. However, the creation of a unified model is complicated by the diversity, heterogeneity and many specific features of projects.

In addition, lender managers often do not have the complete and accurate information needed to assess risks. The available data does not obey the laws of randomness, since the conditions for the implementation of almost every project are unique. There are no statistical patterns here. Therefore, risk assessment methods should take into account their non-statistical nature.

It is also necessary to take into account the heterogeneity of risk sources. Some of the risks are related with the borrower, in particular with the history of relations with him, with the nature of his business, with his financial condition and other sources. Often these risks can only be described by qualitative criteria. We will call qualitative risks such risks that reflect the uncertainty of qualitative magnitudes.

Another part of the risks is related with quantitative magnitudes, in particular with cash flows, which describe the incomes and expenses of the project. Standard economic calculations represent cash flows as standard numbers and use standard arithmetic operations. To account for quantitative risks, managers can use several scenarios for combining project implementation conditions. As a rule, these are pessimistic, optimistic and realistic scenarios. However, this approach has two significant drawbacks. First, scenario calculations, as a rule, do not assess the possibility of implementing scenarios in practice. This somewhat devalues the results, as it does not provide necessary data for decision making. Secondly, if the results of the pessimistic and optimistic scenarios are identified with interval fuzzy numbers (Klir, 1997), then, as studies show (Liu & Guo, 2007; Stefanini et al., 2008;), the difference between the minimum and maximum estimates will increase sharply along with the increase in the number of operations . In the pessimistic scenario, the worst value of each magnitude involved in the calculation is combined only with the worst values of the other magnitudes. This pushes the calculation result far into the area of negative values. The optimistic scenario behaves in a similar way - the result of the calculations rushes into the area of positive values. A large difference between the results of scenario calculations makes it difficult to make a decision, since it increases uncertainty rather than reduces it.

Therefore, based on the Zadeh (1975) extension principle, the most appropriate solution would be to use fuzzy numbers and algorithms for arithmetic operations that constrain the expansion of the carrier of the resulting fuzzy number (this algorithm was developed by Sveshnikov & Bocharnikov (2022) earlier). Then the various characteristics of fuzzy numbers that are used for defuzzification will describe various aspects of PLRs and, due to this, will provide the most complete and high-quality data for assessing project performance, project risks and deciding on project lending.

We pay attention to what is necessary to distinguish between project performance criteria and criteria of quantitative PLRs. We will use standard criteria as project efficiency criteria - numerical estimates of net present value (NPV), payback period (PP) and internal rate of return (IRR). As criteria for quantitative PLRs, we will use the characteristics of fuzzy numbers that describe the criteria for project performance. This understanding is consistent with the standard definition (Purdy, 2010) of risk. Accordingly, we will call quantitative risks such risks that reflect the uncertainty of quantitative magnitudes.

Thus, the aim of our study is to develop procedures for assessing both qualitative and quantitative PLRs under non-statistical uncertainty.

To assessing qualitative PLRs, we propose to use a hierarchical system of criteria, in which the importance of the criteria is described using a Sugeno fuzzy measure, and the generalized estimate is calculated using the Sugeno fuzzy integral. This provides a comprehensive and objective assessment of the level of qualitative risks. To assessing quantitative PLRs, we propose to use the characteristics of fuzzy numbers which describe the project performance criteria and have an arbitrary-form membership function. To perform arithmetic operations, we propose to use the fuzzy arithmetic algorithm developed by us, based on the principle of maximum entropy. The combined use of assessments from both qualitative and quantitative PLRs provides the most complete data for making decisions about lending to small and medium business projects.

2. LITERATURE REVIEW

Keshk et al. (2018) presented the overall concept of risk management. As Wnuk-Pel (2014) points out, most companies continue to use standard decision-making methods based on NPV, scenario analysis and formalization of investment assessing. An extensive critical review of classical strategies is presented in the paper of Kengatharan (2016). Classical decision-making strategies for project lending involve comparing the profitability and risks of the project with some standard values that are established by the company's management. The most common profitability criteria are NPV, PP and IRR. Special indices can also be used as standard criteria, such as the Sharpe (1966) ratio or Roy's (1952) safety criterion, which additionally take into account the risk of loss of profitability. These risks are determined by comparing the profitability of the project with the profitability of risk-free assets, or are determined as a deviation from the profitability of other assets.

A popular way to improve classical methods is to use the Zadeh (1975) extension principle, according to which standard values of magnitudes can be replaced by fuzzy values. In particular, cash flows can be represented as fuzzy numbers, and their calculation can be performed using fuzzy arithmetic.

In the article, Łyczkowska-Hanćkowiak (2020) proposes using oriented trapezoidal fuzzy numbers to represent various criteria for assessing the financial performance, in particular: Sharpe, Sortino, Jensen and others ratios.

Wójcicka-Wójtowicz and Piasecki (2021) also propose to use oriented fuzzy numbers to solve the problem of classifying potential debtors, in particular, to determine the numerical order scale and scoring.

Another use of oriented fuzzy numbers is presented in the article by the authors Pisz et al. (2019), who propose to assess cash flows and profitability of investment projects using oriented fuzzy numbers.

In the paper, Gejirifu et al. (2019) propose the use of intuitionistic fuzzy sets to assess the credit risk of electricity trading companies. These sets make it possible to achieve more accurate subjective judgments and thereby more accurately describe the current state of the company's creditworthiness.

To represent the cash flows and NPV of projects, Appadoo et al. (2008) propose using fuzzy numbers with LR approximation by power functions.

Note that one of the main problems of fuzzy arithmetic is the problem (Mareš, 1997) of ensuring the equivalence of standard and fuzzy arithmetic. This so-called “fuzzy zero and fuzzy one” problem. Oriented fuzzy numbers ensure compliance with the axiomatic of standard arithmetic operations; however, additional expert information is required to define such numbers. Other problems of fuzzy arithmetic remain, in particular it's the problem (Bede & Fodor, 2006) of preserving the form of the membership function when multiplying fuzzy operands. When using triangular membership functions, there is a problem (Kosheleva et al., 1997) of information loss. The most important problem (Liu & Sizong, 2007; Stefanini et al., 2008) of fuzzy arithmetic is the sharp increase in the carrier of the resulting fuzzy number in the case of multiple operations. This problem leads to an "explosive" increase in uncertainty and a result that cannot be rationally interpreted.

To select investments according to the criterion of NPV, Lesage (2001) proposed using fuzzy relations instead of arithmetic operations with fuzzy numbers, the implementation of which does not require compliance with the strict axiomatic of arithmetic operations, what makes it possible to weaken the above problems.

Another way to determine the risks of projects is to find the closest analogues of the project in the database of previously implemented projects or in the knowledge base, which contains information about the relationships between the profitability of previously implemented projects and other factors. One variant of this approach is determining the proximity of the project to the prototypes of some classes of previously implemented projects in order to select the closest class. Such methods can be called methods of implicit extrapolation, since they involve the projection of the conditions of the past onto the future conditions of the project. It should be noted that in the case of rapid changes in the financial and economic environment, existing databases and knowledge bases quickly become outdated. When processing them, it is necessary to give priority to more recent knowledge, what is not always provide.

In many credit risk assessment problems, researchers establish a constant threshold for the difference between classes of good and bad loans. Jaya and Tamilselvi (2018) propose using a variable threshold to improve classification efficiency. The choice of the threshold value is based on the evaluation criteria of the data set and the classifier.

Namvar and Naderpour (2018) propose to use the Choquet integral to join of risk assessment results obtained on the base of multiple classifiers. According to the authors, this makes it possible to improve the accuracy of creditworthiness assessment in P2P lending.

In the article, Sirbiladze et al. (2010) propose to consistently use the “experton” method and possibilistic discriminant analysis to minimize risks by choosing the best investment projects. The first method is used for the initial selection of projects with minimal risk. It is based on replacing a single factor estimate with a possibility interval. The second method is used to compare and sort the selected projects. It is based on the construction of a tabular-numerical knowledge base, which contains expert assessments about the possible dependence of the correct solution and some factor.

Ghatasheh (2014) proposes to use a Random Forest Trees algorithm with parameter tuning to predict a borrower's credit risk based on data processing in the German Credit dataset.

Another way to determine the risks of projects is a multi-criterion assessing. This assessing is in the weighting of partial attributes and the calculation of the generalized estimation. Most often, this method is used in individual lending. For example, in the article, Li (2015) proposes to assess individual credit risk using Saaty's analytical hierarchical process in conjunction with Delphi's group decision making.

Thus, in the modern scientific literature, the problem of joint assessment of both qualitative and quantitative PLRs is insufficiently reflected and, therefore, continues to be relevant.

3. METHODOLOGY AND THE RESEARCH PROBLEM

In this section, we have presented the approaches proposed for risk assessment. We have shown above that the nature of risks divides them into two types: qualitative and quantitative. Therefore, below we will consider separately the corresponding methods of assessment.

3.1. Evaluation of qualitative PLRs

To assessing qualitative risks, we can use the widespread multi-criteria decision-making (MCDM) methods, of which there are a lot of known today. In order to choose the method that best suits our problem, we will look at how the mathematical constructions of these methods take into account non-statistical uncertainty. Various MCDM methods have approximately the same scheme for evaluating the object, which is the lending project.

Object evaluation scheme. The project can be described using a set of partial criteria. The value of each criterion can be established on a discrete set of gradations. For example, the partial criterion “Existence time of the borrower's company” can be established on a set of gradations: {up to 1 year, from 1 to 5 years, more than 5 years}. Gradations can also be formulated in terms of desirability: {bad, satisfactory, good}. The accuracy of evaluating the partial criterion and the sensitivity of the result will depend on the number of gradations. The evaluating process consists in the selection of one or more gradations. After evaluating all partial criteria, the MCDM algorithm generalizes partial estimates taking into account the importance of the criteria, the combination of which can be a complex multi-stage hierarchy with one resulting criterion. The estimation in this criterion is a generalized estimation of the qualitative PLR. The adequacy of this estimation directly depends on the choice of the method used for aggregating partial estimates.

For aggregating we propose to use the Sugeno fuzzy integral (Sugeno, 1972) *(s)*∫ from the fuzzy membership function h:X→[0,1], defined on the discrete set $A \in X, X = \{x_i,\ i = \overline{1,N}\}$, along the fuzzy measure $g(\cdot)$ is represented as follows:

$$(s)\int_A h(x) \circ g(\cdot) = \max_{i=1,N}\Big(min\big(h(x_i), g(H_i)\big)\Big) \quad (1)$$

where $H_i = \{x_j \mid h(x_j) \geq h(x_i),\ j = \overline{1,N}\}$.

The Choquet fuzzy integral (Choquet, 1954) can also be used to aggregate partial estimates. The results of calculations of these integrals practically do not differ from each other. The Sugeno integral has useful result insensitivity which models the natural indifference of the resulting estimate to minor changes in the input data. In addition, the technique for calculating the Sugeno integral allows you to select from the set of partial criteria those criteria that influenced the resulting estimation, that is, the use of the Sugeno integral allows you to provide additional information for decision making. In what follows, we will use the Sugeno fuzzy integral, since it is more convenient in practice.

The main useful property between the fuzzy integral (both Sugeno and Choquet) is the dependence of the properties of the integration result on the λ-parameter of the fuzzy measure. The λ-parameter determines the modality of the fuzzy measure, that is, the relation of the content of the statement to reality (Kaufmann et al., 2006). For example, if λ>0, the fuzzy measure is a superadditive or confidence measure. If λ□0, the fuzzy measure is the measure of necessity. If -1<λ<0, the fuzzy measure is a subadditive or plausibility measure. If λ=-1, the fuzzy measure is a possibility measure. If λ=0, the fuzzy measure is a probability measure. As shown by Averkin et al. (1986), a fuzzy measure is a parametric extension of a probability measure. Therefore, the most common aggregation method – the weighted average – can be considered as a special case of the fuzzy integral. Another special case – max min – corresponds to the case λ=-1.

It follows from this that the Sugeno fuzzy integral is a more flexible tool for assessing qualitative risks. In particular, by changing the λ-parameter of the fuzzy measure $g(\cdot)$, we will change the risk assessment logic. For example, we built a fuzzy measure of the criteria importance with λ=-1 (the fuzzy measure of possibility). Then the resulting estimation will be maximal in the case when the estimation of at least one partial criterion will be maximal. Such logic can conditionally be called “minority logic”. Conversely, if $\lambda \to \infty$ (fuzzy measure of necessity), then the resulting estimation will be maximal only if the estimations of all partial criteria will be maximal. Such logic can conditionally be called “the logic of the majority”.

Thus, the advantage of using the Sugeno fuzzy integral is to provide flexibility in the procedure for aggregating partial criteria. This flexibility is necessary because different parts of the criteria hierarchy may require different aggregation logic. Other aggregation methods do not provide these properties.

3.2. Assessing of quantitative PLRs

We emphasize that we are considering the project from the view-point of the lender, who is interested not only in profit, but also in guarantees of repayment of loan funds within the prescribed period. Therefore, we must provide the lender with an assessment of the economic performance of the project and an assessment of the quantitative risks of the project. As we indicated above, to assess the economic performance of the project, we will use the classical approaches that are accepted in most credit organizations. To assess quantitative risks, we will use fuzzy numbers as a tool to describe the uncertainty that is contained in the criteria of the economic efficiency of the project.

The procedure for calculating these criteria is standard. The most generalized criterion of the project performance is NPV. This criterion is calculated using incomes and expenses flows. Other widely used project performance criteria are the payback period (PP) and the internal rate of return (IRR). Both criteria are derived from NPV.

We define quantitative PLRs based on the characteristics of fuzzy numbers that describe the values of project performance criteria. For example, we consider the deviation of NPV from the most expected value as one of the main criteria for quantitative PLR. The more possible deviations in the direction of decreasing NPV, the greater this risk. And vice versa, the greater the possible deviations in the direction of increasing NPV, the greater the resistance of the project to possible adverse circumstances.

As shown in the literature review, there are many variants for describing numerical uncertainty. Each variant has its own individual advantages and disadvantages. We propose to use discretized fuzzy numbers with an arbitrary form of the membership function and algorithm of fuzzy arithmetic based on the maximum entropy principle. As we showed in a special study (Sveshnikov & Bocharnikov, 2022), these methods are the most suitable for solving applied problems.

Let's consider the representation of discretized normal (unimodal) fuzzy numbers of arbitrary form and their characteristics, which will be important for assessing quantitative PLRs. We will call as fuzzy number $\tilde{R}$ a fuzzy set with support defined on the set of real numbers:

$$\tilde{R} = \{(x, y) \in \mathbb{R} \times [0,1]: y = \mu_{\tilde{R}}(x)\},$$

$$\mu_{\tilde{R}}(x) = \begin{cases} f_{\tilde{R}}(x) & \text{when } x \in [a, b), \\ 1 & \text{when } x \in [b, c], \\ g_{\tilde{R}}(x) & \text{when } x \in (c, d] \\ 0 & \text{otherwise,} \end{cases}$$

where $\mu_{\tilde{R}}(x)$ is the membership function, $a, b, c, d \in \mathbb{R},\ a \le b \le c \le d,\ f_{\tilde{R}}(\cdot)$ is an increasing right continuous function and $g_{\tilde{R}}(\cdot)$ is a decreasing left continuous function.

To represent a fuzzy number of arbitrary forms in a computer, we use discretization along the abscissa axis, on which the carrier of the fuzzy number is given. In this case, the fuzzy number is represented as a set of pairs:

$$\tilde{R} = \left\{ \begin{matrix} (x_i, \mu(x_i)),\ i = \overline{1, n}, \\ x_{i+1} - x_i = \dfrac{\mathrm{d} - \mathrm{a}}{n - 1},\ x_0 = a,\ x_n = d \end{matrix} \right\} \quad (2)$$

where *n* is the number of discrets (segments), *d*, *a* are the lower and upper boundaries of the carrier of the fuzzy number.

Today, many characteristics of fuzzy numbers are known, the choice of which depends on the needs of applied research. A fairly complete review of these characteristics is presented in the work (Bodjanova, 2005). Figure 1 illustrates the most common characteristics of fuzzy numbers that are used to assess quantitative risks.

1. The minimum at the α-level is the segment of the carrier, to the left of which values of the membership function of any segments are less:

$$x^{\alpha min} = \min\{x_i | \mu(x_i) \geq \alpha \in [0,1], x_i \in R\} \quad (3)$$

2. The maximum at the α level is the segment of the carrier, to the right of which the values of the membership function of any segments are less:

$$x^{\alpha max} = \max\{x_i | \mu(x_i) \geq \alpha \in [0,1], x_i \in R\} \quad (4)$$

3. The most expected value of a fuzzy number is the segment of the carrier, which has the biggest value of the membership function:

$$x^{MEV} = \left\{ x_i | \mu(x_i) = \max_{j=\overline{1,n}} \mu(x_j), x_i \in R \right\} \quad (5)$$

4. The center of gravity of a fuzzy number is the segment of the carrier, to the right and left of which the areas of the figures restricted by membership function are equal:

$$x^{CG} = \sum_{i=1}^{j-1} |x_i - x_{i+1}| \cdot \mu(x_i), j = \overline{1,n}, x^{CG} \in R \quad (6)$$

5. Risk-function of a fuzzy number:

$$\rho(x_k) = \min_{x_i \geq x_k} \mu(x_k) - \max_{x_i \leq x_k} \mu(x_k), \quad i,k = \overline{1,n} \quad (7)$$

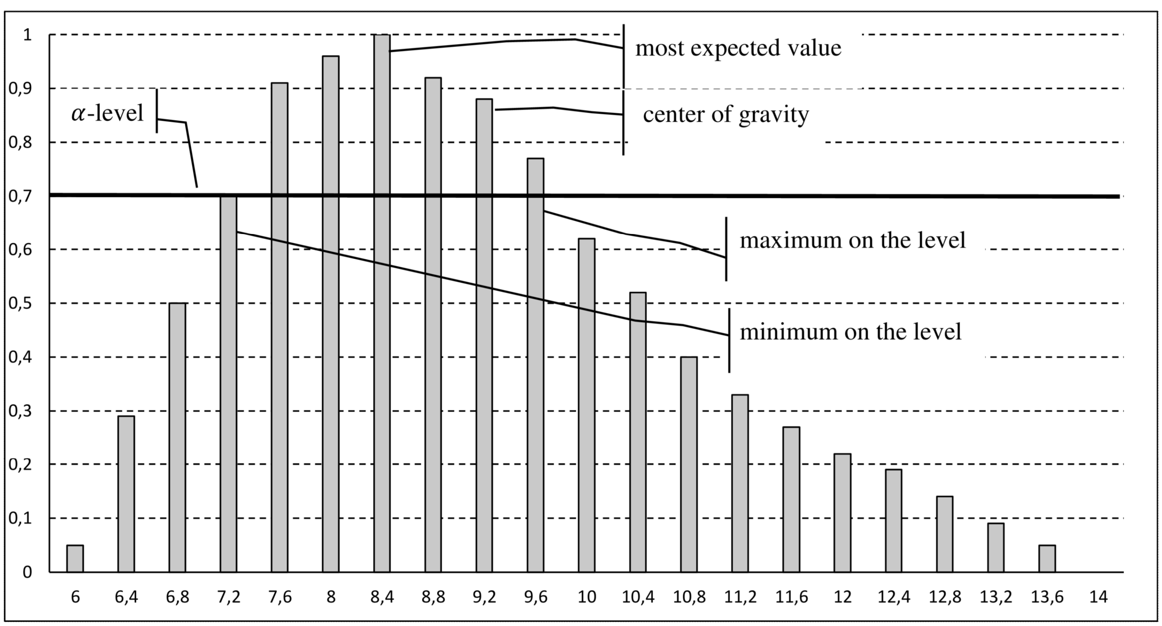

Figure 1. Characteristics of fuzzy numbers

We proposed this new characteristic because we encountered a contradiction in assessing the quantitative risks of various projects, which we will discuss below.

The risk-function of the fuzzy number $\tilde{R}$ describes the possibility that in reality the value of the numerical magnitude will be greater than the value that describes the fuzzy number $\tilde{R}$. On Figure 2 shows the risk-function of the fuzzy number, which is shown in Figure 1.

Figure 2 shows two areas: the risk area and the anti-risk area (resilience area). To understand the meaning of these areas, consider an example of using the risk-function. In Figure 2, we have highlighted by shading two segments of the fuzzy number, which describe the profit of a project: (x_i=10.8; $\mu(x_i)$=-0.6) и (x_k=7.2; $\mu(x_k)$=0.3). Let's assume that we are interested in increasing the profit of the project. Then the first segment means the following. If, when making a decision to lend a project, we will focus on a profit equal to x_i=10.8, then the risk of making a wrong decision will be equal to $\mu(x_i)$=0.6. The second segment means that if we focus on profit equal to x_k=7.2, then there will be no risk of making a wrong decision, and the possibility of a correct decision will be equal to $\mu(x_k)$=0.3.

Thus, the risk-function directly describes two ontological characteristics of risk: the size of possible losses or gains, and also characterizes the occurrence of these events in terms of possibility. The use of the risk-function provides managers with additional data for making decisions regarding project lending.

We will remind that our problem consists of two parts.

Subproblem 1. We have to develop an algorithm for assessing a generalized qualitative PLR based on MCDM-method using a Sugeno fuzzy measure to describe the importance of criteria and a Sugeno fuzzy integral to generalize partial estimations of criteria taking into account their importance. To develop this algorithm, it is necessary to determine:

-hierarchy of criteria for assessing

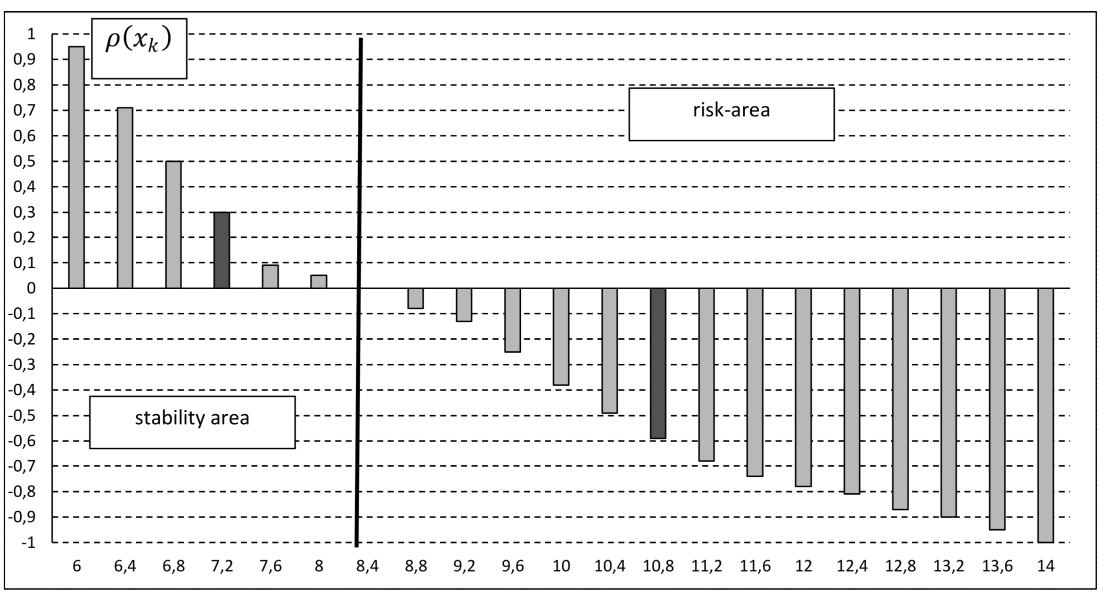

Figure 2. Risk-function of the fuzzy number, which is shown in Figure 1

qualitative risk;

-the importance of these criteria;

-scale and procedure for estimating initial criteria (estimation can be performed by several experts);

-procedure for calculating the generalized qualitative PLR.

Subproblem 2. We have to develop an algorithm for assessing the generalized quantitative PLR based on the project performance criteria (NPV, PT and IRR), which are described using fuzzy numbers.

4. RESULTS

4.1. Algorithm for assessing a generalized qualitative PLR

Hierarchy of criteria for assessing qualitative risk.

Determining the hierarchy of criteria for assessing qualitative risk involves the definition of criteria, their physical meaning and interrelations. Ensuring completeness is the main requirement for the criteria hierarchy. Here we must not overlook the important details on which qualitative PLRs depend. As a result of considering the nature of qualitative PLRs and their relationships, we propose the following hierarchy of criteria, which is shown in Figure 3.

The hierarchy is a criteria combination which characterize partial qualitative risks and their interrelations. When constructing the hierarchy, we proceeded from the following empirical requirements.

1. To ensure the completeness of the assessment of the generalized qualitative risk, the borrower's project must be considered from all sides, that is, all risks that may affect the implementation of the project and affect the return of borrowed funds should be considered.

2. At the lower level of the hierarchy are located partial criteria that must be measurable. In other words, these criteria should be formulated in such a way that the lender's manager, using the borrower's documentation, can evaluate the project in these criteria. We will call these criteria initial criteria.

3. Partial criteria of the lower level are the area of definition of the criteria, which are located at a higher level of the hierarchy. In other words, partial estimations should be taken into account in more generalized estimations. A criterion of a higher level can simultaneously be a partial criterion. In this case, we will call it the intermediate criterion.

4. Partial criteria that are associated with the same generalized criterion must have a single logical basis and must have the same level of system generalization. In one criterion, you cannot mix assessments, for example, of management and the borrower's credit history, since these are disparate categories.

5. The generalized criterion of qualitative PLR is located at the top level of the hierarchy. This criterion includes all partial and intermediate criteria with taking into account their importance and place in the hierarchy. We also tried to ensure that the proposed hierarchy could be used in many areas of production of goods and services.

Let's consider further short explanations of all criteria from this hierarchy.

The generalized criterion J of the qualitative PLR characterizes the full set of qualitative risks that may affect the return of borrowed funds.

The intermediate criterion x_1 “borrower's company” characterizes the totality of risks that are associated with the borrower's company as an institution (enterprise).

The initial criterion x_{11} "the duration of the company work" characterizes the risk associated with deliberate bankruptcy or the inexperience of the borrower's company. The older the company, the lower the risk.

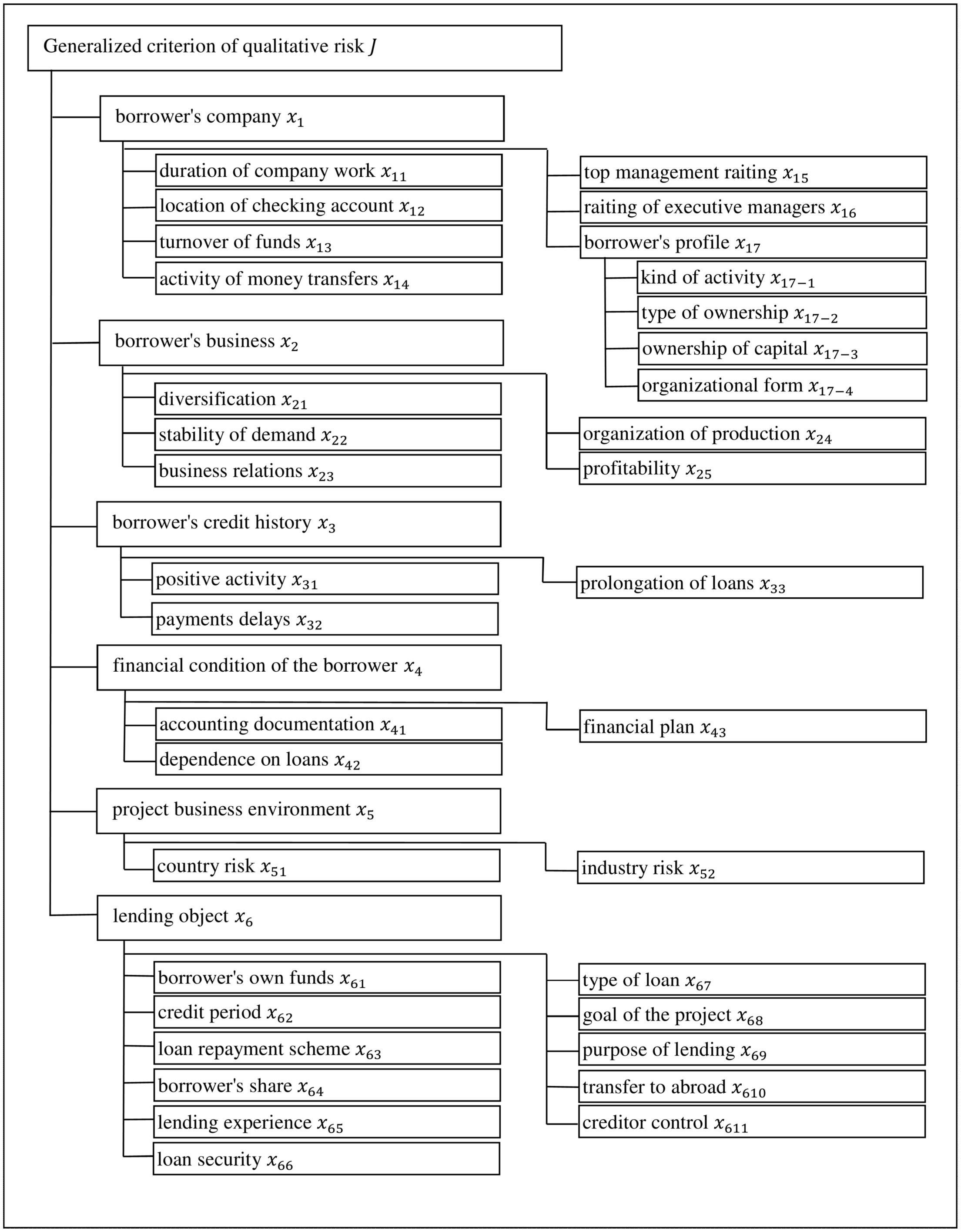

Figure 3. Hierarchy of criteria for assessing qualitative risk

The initial criterion x_{12} “location of checking account” characterizes the risk associated with the possibility of controlling the borrower's spending of credit funds. As a rule, lenders lend to clients with confidence if they have a checking account with the lender's bank.

The initial criterion x_{13} “turnover of funds” characterizes the risk associated with insufficient turnover of the borrower's capital. The higher the turnover, the lower the risk.

The initial criterion x_{14} “activity of money transfers” characterizes the risk associated with the instability of cash receipts to the borrower's checking account. The more often the funds are transferred, the lower the risk.

The initial criterion x_{15} “top management raiting” characterizes the risk associated with the potential quality of project management. This risk depends on the education, training, experience, constructiveness, integrity and other business qualities of the borrower's top managers. Trained specialists inspire confidence in the positive outcome of the project. Therefore, the better the specialists in the borrower's team, the lower the risk.

The initial criterion x_{16} “raiting of executive managers” characterizes the risk associated with the efficiency and quality of the project implementation. This risk depends on the general level of training of executive managers and executive discipline in the company of the borrower. Good executive discipline in the borrower's company gives the lender confidence that all decisions will be implemented in a quality and timely manner. Therefore, the better the executive managers, the lower the risk.

Intermediate criterion x_{17} “borrower's profile” characterizes the totality of risks that can be identified from the classification features of the borrower's company. These features can show the prospects of the borrower, indirectly characterize the conditions for repaying the loan in the case of the borrower's bankruptcy, and characterize the difficulty of the borrower's control from the part of the lender.

The initial criterion $x_{17\text{-}1}$ “kind of activity” characterizes the risk associated with the type of activity of the borrower's company: production, scientific testing, construction, transport, agriculture, trade, and others.

The initial criterion $x_{17\text{-}2}$ “type of ownership” characterizes the risk that is associated with the form of ownership of the borrower's company: state, municipal, private, property of public organizations, mixed.

The initial criterion $x_{17\text{-}3}$ “ownership of capital” characterizes the risk that is associated with the country under whose jurisdiction the capital of the borrower's company is located: national, foreign, mixed.

The initial criterion $x_{17\text{-}4}$ “organizational form” characterizes the risk that is associated with the form of organization of the borrower's company: joint-stock company, limited liability company, additional liability company, general partnership, unitary enterprise and others.

The intermediate criterion x_2 “borrower's business” characterizes the totality of risks that are associated with the business of the borrower's company.

The initial criterion x_{21} “diversification” characterizes the ability to compensate for losses in some business of the borrower at the expense of incomes from another business of the borrower. If the borrower does business in several areas, then in case of failure of the project, he can repay the loan at the expense of incomes from other projects.

Therefore, the greater diversification of the borrower's business, the lower the risk.

The initial criterion x_{22} “stability of demand” characterizes the stability of demand for the borrower's products. If the demand for the borrower's products is stable and does not fluctuate, it can be assumed that the borrower can easily accumulate cash and pay expenses without delay in case of unexpected expenses. Therefore, the greater the stability of demand for the borrower's products, the lower the risk.

The initial criterion x_{23} “business relations” characterizes the risk associated with the instability of the borrower's relations with suppliers and buyers of products. Preference of any lender should be given to borrowers with well-established business relationships. Therefore, the greater the stability of the borrower's business relations, the lower the risk.

The initial criterion x_{24} “organization of production” characterizes the risk associated with the ability of the borrower to organize the production of products, in particular: ensure the smooth running of production processes, ensure the consistency of production plans, minimize downtime of production facilities, and so on. Well-established production processes have a positive effect on the lender's confidence in the repayment of the loan. Therefore, the better the borrower's production is organized, the lower the risk.

The initial criterion x_{25} “profitability” characterizes the risk associated with the low profitability of the borrower's business. The higher profitability, the lower the risk.

Intermediate criterion x_3 “borrower's credit history” characterizes the totality of risks that are associated with the relationship between the lender and the borrower in the past.

The initial criterion x_{31} “positive activity” characterizes the risk associated with uncertainty about the borrower's good behavior after obtaining a loan. We can evaluate the level of this risk based on the positive experience of lending in the past. As a rule, lenders treat casual clients with distrust. Therefore, the more positive the borrower's lending experience, the lower the risk.

The initial criterion x_{32} “payments delays” characterizes how often the borrower delayed payments on loans to the lender or other lenders. The presence of delays in payments in the history of the borrower increases the risk of project implementation.

The initial criterion x_{33} “prolongation of loan” characterizes how often the borrower prolonged the loans due to problems in project implementation. Frequent loan extensions in the past may indicate increased risks in the future.

The intermediate criterion x_4 “financial condition of the borrower” characterizes the totality of risks associated with the financial condition of the borrower.

The initial criterion x_{41} “accounting documentation” characterizes the risks that can be identified based on the study of accounting documents (balance sheets, cash flow statements, equity statements). The quality of accounting documents should also be taken into account in this criterion.

The initial criterion x_{42} “dependence on loans” characterizes the risk of a borrower's increased dependence on loans, which is determined on the basis of a study of accounting documents. The greater the borrower's dependence on loans, the higher the risk.

The initial criterion x_{43} “financial plan” characterizes the risk associated with the confidence that the future incomes of the borrower will allow the loan to be repaid. The quality of the financial plan should also be considered in this criterion.

Intermediate criterion x_5 “project business environment” characterizes the totality of country and industry risks.

The initial criterion x_{51} “country risk” characterizes the risk associated with the future actions of the country in which the project is planned to be implemented. These actions may affect the borrower's ability to meet its obligations to the lender. The area of country risk primarily includes changes in exchange rates, international sanctions, and the like.

The initial criterion x_{52} “industry risk” characterizes the risk associated with possible changes in industry conditions that may affect the implementation of the project. The area of industry risk primarily includes competition conditions, substitute products, new technologies, and the like.

Intermediate criterion x_6 “lending object” characterizes the totality of risks associated with the borrower's project, which is the object of lending.

The initial criterion x_{61} “borrower's own funds” characterizes the risk that the borrower will not be able to fully or partially repay the loan using their own funds. The ratio of the borrower's own funds to the amount of the loan shows the level of this risk. The higher the ratio, the lower the risk.

The initial criterion x_{62} “credit period” characterizes the risk of underestimation or overestimation of the crediting period by the borrower. The matching of the loan period and the nature of the project shows the level of this risk. If, in the opinion of the lender, the loan period is too high or too low, this may raise suspicions about the true intentions of the borrower.

The initial criterion x_{63} “loan repayment scheme” reflects the risks associated with the borrower's proposed repayment procedure. For example, repaying the entire loan amount at the end of the loan period allows the borrower to better concentrate the funds, but is more risky for the lender.

The initial criterion x_{64} “borrower's share” characterizes the risk associated with the borrower's disinterest in incurring losses at his own expense in case of project failure. Projects in which the share of the borrower in the total amount of financing is high can be assessed as less risky.

The initial criterion x_{65} “lending experience” characterizes the risk associated with the fact that the lender does not have sufficient experience in lending to similar projects. Insufficient lending experience increases the risk for the lender.

The initial criterion x_{66} “loan security” characterizes the risk associated with the quality of guarantees, thanks to which the lender obtains confidence in the repayment of the loan. The worse the guarantees, the higher the risk.

The initial criterion x_{67} “type of loan” characterizes the risk associated with the conditions for the implementation of operations to provide the borrower with funds within the framework of the loan. As these operations can be considered: transfer of funds to the checking account of the borrower, overdraft, factoring and others.

The initial criterion x_{68} “goal of the project” characterizes the risk associated with the goal of the project and with the potential success of its implementation. These goals are usually determined by the general policy of the lender. For example, venture projects can be considered as the most risky ones. Projects that aim to expand production may have lower risks.

The initial criterion x_{69} “purpose of lending” characterizes the risk associated with the purpose for which the borrower is applying for a loan. This goal may indirectly characterize the actual state of the borrower's business, as well as guarantees for the repayment of the loan. For example, if the goal is to purchase equipment, then such a loan can be considered less risky, since if the project fails, the equipment can be sold and the loan returned.

The initial criterion x_{610} “transfer to abroad” characterizes the risk associated with the transfer of credit funds abroad. Projects that are fully or partially implemented abroad or with the participation of foreign counterparties have increased risks, since these projects complicate control by the lender.

The initial criterion x_{611} “creditor control” characterizes the risk that is associated with the inability to control the progress of the project on the part of the lender. If the terms of the loan are defined in such a way that the lender can control the cash and commodity flows, then such a project can be assessed as less risky.

The presented hierarchy contains 35 initial criteria and 7 intermediate criteria. The list of criteria and their interrelationships can be changed depending on the lender's priorities and the specifics of its business. Explanations of these criteria can also be supplemented and clarified in instructions for the management of the lender.

The importance of criteria for assessing qualitative risk.

We will make some notation which will be used below. The main criterion J is located at the first level of the hierarchy. Let's denote the set of partial criteria of the second level of the hierarchy as A=$\{x_1,x_2,\ldots,x_6\}$. These criteria are subordinate to the main criterion J and are its domain of definition. Each criterion of the second level from A has its own domain of definition, which consists of criteria of the third level:

$B_1=B_{11}\cup B_{12}, B_{11}=\{x_{11},x_{12},x_{13},x_{14},x_{15},x_{16}\}, B_{12}=\{x_{17}\}$ are subordinate to the criterion x_1;

$B_2=\{x_{21},x_{22},x_{23},x_{24},x_{25}\}$ are subordinate to the criterion x_2;

$B_3=\{x_{31},x_{32},x_{33},x_{34}\}$ are subordinate to the criterion x_3;

$B_4=\{x_{41},x_{42},x_{43}\}$ are subordinate to the criterion x_4;

$B_5=\{x_{51},x_{52}\}$ are subordinate to the criterion x_5;

$B_6=\{x_{61},x_{62},x_{63},x_{64},x_{65},x_{66},x_{67},x_{68},x_{69},x_{610},x_{611}\}$ are subordinate to the criterion x_6.

Set B_1 is divided into two sets depending on whether the criterion is initial or not. All third level criteria from B_{11} are initial. The only criterion of the third level from B_{12} must be defined on the set of criteria of the fourth level $C_{17}=\{x_{17\text{-}1},x_{17\text{-}2},x_{17\text{-}3},x_{17\text{-}4}\}$. Criteria from set C_{17} are also initial. Thus, we denote the set of initial criteria as $E = B_{11} \cup \bigcup_{i=2}^{i=6} B_i \cup C_{17}$, and the set of the rest as $D=A\cup B_{12}$. Recall that the estimations of PLRs in the initial criteria must be set by the manager based on the study of the project documentation. The estimations of the PLRs in the remaining criteria should be calculated based on the generalization of the estimations in the subordinate criteria.

As shown above, fuzzy measures are the most appropriate way to describe the importance of criteria. To build fuzzy measures, we used the successive approximation method described in paper (Takahagi, 2000). This method assumes that the manager forms an initial approximation for a fuzzy measure using paired comparisons (Saaty & Kearns, 1985) and iteratively changes this measure until the λ parameter becomes equal to the required value. These measures are shown in Table 1.

Scale and procedure for estimating initial criteria.

For ease of use, we reindex the set E. Denote $e_\beta \in E$ as an element of the set E, where the index $\beta = \overline{1,\ 35}$ reindexes the elements of the set E, as shown in Table 2.

Denote the set of managers as $F = \{f_j,\ j = \overline{1,\ M}\}$, where M is the number of managers. In accordance with the work (Ayyub & Klir, 2006), the scale and procedure for estimating initial criteria are designed to determine by each manager of values $\varepsilon(e_\beta|f_j)$ of criteria $e_\beta \in E$. We have built a evaluating scale based on the Harrington (1965) "desirability" curve, which is shown in Figure 4.

The range of this curve values is divided into five intervals that have linguistic estimations, as shown in Table 3.

The estimating procedure is as follows. To obtain an estimate of $\varepsilon(e_\beta|f_j)$, the manager f_j must study the project documentation and select from Table 3 the linguistic value k that best matches the manager's opinion. Then the manager must choose a numerical estimate

Table 1. Fuzzy measures of criteria importance

Generalized criteria	J										
Domain of definition (the set A)	x_1	x_2	x_3	x_4	x_5	x_6					
Fuzzy measures of importance $g_A(\cdot)$	0.097	0.129	0.075	0.172	0.172	0.258					
Intermediate criteria	x_1										
Domain of definition (the set B_1)	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}				
Fuzzy measures of importance $g_{B_1}(\cdot)$	0.11	0.385	0.495	0.264	0.2	0.187	0.31				
Intermediate criteria	x_2										
Domain of definition (the set B_2)	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}						
Fuzzy measures of importance $g_{B_2}(\cdot)$	0.362	0.926	0.472	0.65	0.75						
Intermediate criteria	x_3										
Domain of definition (the set B_3)	x_{31}	x_{32}	x_{33}								
Fuzzy measures of importance $g_{B_3}(\cdot)$	0.361	0.847	0.626								
Intermediate criteria	x_4										
Domain of definition (the set B_4)	x_{41}	x_{42}	x_{43}								
Fuzzy measures of importance $g_{B_4}(\cdot)$	0.6	0.85	0.75								
Intermediate criteria	x_5										
Domain of definition (the set B_5)	x_{51}	x_{52}									
Fuzzy measures of importance $g_{B_5}(\cdot)$	0.766	0.479									
Intermediate criteria	x_6										
Domain of definition (the set B_6)	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}	x_{67}	x_{68}	x_{69}	x_{610}	x_{611}
Fuzzy measures of importance $g_{B_6}(\cdot)$	0.5	0.5	0.35	0.55	0.45	0.65	0.11	0.22	0.25	0.68	0.7
Intermediate criteria	x_{17}										
Domain of definition (the set C_{17})	x_{17-1}	x_{17-2}	x_{17-3}	x_{17-4}							
Fuzzy measures of importance $g_{C_{17}}(\cdot)$	0.11	0.37	0.41	0.35							

Table 2. Reindexing elements of the set E

x_{11}	→	e_1	x_{12}	→	e_2	x_{13}	→	e_3	x_{14}	→	e_4	x_{15}	→	e_5	x_{16}	→	e_6
x_{21}	→	e_7	x_{22}	→	e_8	x_{23}	→	e_9	x_{24}	→	e_{10}	x_{25}	→	e_{11}	x_{31}	→	e_{12}
x_{32}	→	e_{13}	x_{33}	→	e_{14}	x_{34}	→	e_{15}	x_{41}	→	e_{16}	x_{42}	→	e_{17}	x_{43}	→	e_{18}
x_{51}	→	e_{19}	x_{52}	→	e_{20}	x_{61}	→	e_{21}	x_{62}	→	e_{22}	x_{63}	→	e_{23}	x_{64}	→	e_{24}
x_{65}	→	e_{25}	x_{66}	→	e_{26}	x_{67}	→	e_{27}	x_{68}	→	e_{28}	x_{69}	→	e_{29}	x_{610}	→	e_{30}
x_{611}	→	e_{31}	x_{17-1}	→	e_{32}	x_{17-2}	→	e_{33}	x_{17-3}	→	e_{34}	x_{17-4}	→	e_{35}			

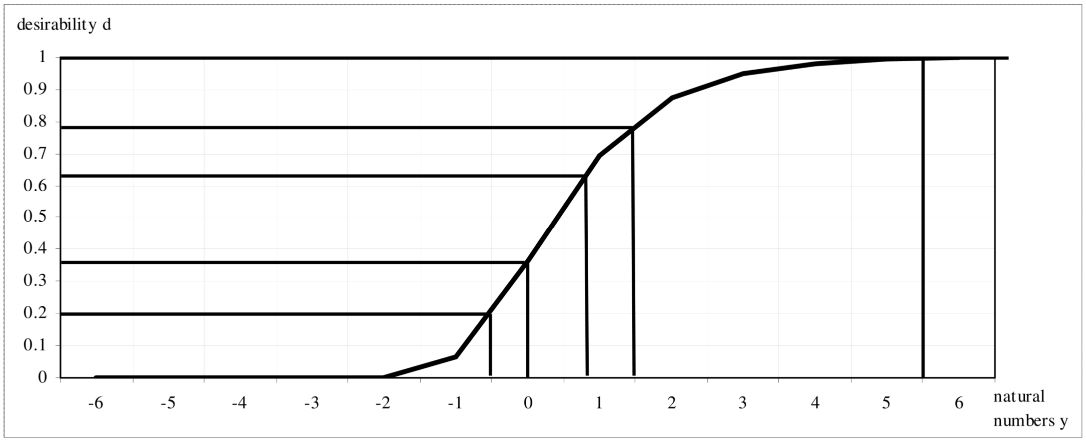

Figure 4. "Desirability" curve of Harrington

Table 3. Evaluating scale

№	Linguistic estimation of the characteristic value	Intervals of Harrington's curve $H_k = \{h_k,\ k = \overline{1,\ 5}\}$
1	Excellent	0.8 – 1
2	Good	0.63 – 0.79
3	Satisfactory	0.37 – 0.62
4	Bad	0.2 – 0.36
5	Very bad	0 – 0.19

$\varepsilon(e_\beta\ |f_j)\in h_k$ from the appropriate interval. This two-step procedure makes it possible to increase the accuracy of reflecting the opinions of managers in numerical estimates and, due to this, also increase the accuracy of measuring the initial criteria estimates.

Procedure for calculating the generalized qualitative PLR.

After the lender's managers have studied the project documents and determined the PLRs' estimations in the initial criteria, the following steps must be performed. The first step provides to obtain estimates of partial PLRs in all initial criteria. The remaining steps provide a generalization of these risks according to their hierarchy.

Step 1. Aggregation of estimates from several managers, taking into account their competence.

For the sake of simplicity, we will assume that each manager has determined all estimates of the PLRs in the initial criteria, although in practice there may be other variants. Managers have different skill levels, which can be a priori described as a fuzzy measure of managers competence in the form $g_F(\cdot){:}2^F\rightarrow[0,1]$. The Sugeno fuzzy integral from the membership function $\varepsilon(e_\beta\ |f_j)$ along the fuzzy measure $g_F(\cdot)$ calculates the group estimate of the PLRs $w(e_\beta)$ in each criterion from the set E:

$$w(e_\beta) = (s)\int_F \varepsilon(e_\beta|f_j) \circ g_F(\cdot) \qquad (8)$$

These estimates will then be aggregated according to a criteria hierarchy of generalized PLR (see Steps 2-4).

Step 2. Aggregation of initial estimates from the set C_{17}.

Here we use fuzzy measure of the importance of the initial criteria from the set C_{17} (see Table 3). This step allows you to obtain estimates of PLRs in all third level criteria:

$$\mu(x_{17}) = (s) \int_{C_{17}} w(e_\beta) \circ g_{C_{17}}(\cdot)$$

Step 3. Aggregation of estimates from the sets $B_k, k = \overline{1,6}$.

Here we use a fuzzy measure of importance of criteria $g_{B_k}(\cdot), k = \overline{1,6}$ (see Table 3). This step allows you to obtain estimates of PLRs in all second level criteria:

$$\mu(x_k|B_k) = (s) \int_{B_k} w(e_\beta) \circ g_{B_k}(\cdot), k = \overline{1,6}$$

Step 4. Aggregation of estimates from set A.

Here we use fuzzy measure of importance of criteria g_A. This step allows you to obtain a generalized estimate of the qualitative PLR:

$$J^{qual} = (s) \int_{x_i \epsilon A} \mu(x_i) \circ g_A(\cdot), i = \overline{1,6} \qquad (9)$$

Thus, the algorithm for assessing a generalized quantitative PLR provides the calculation of the estimates of the generalized qualitative PLR.

4.2. Algorithm for assessing a generalized quantitative PLR

Recall that we must develop an algorithm for assessing the generalized quantitative PLR based on the standard NPV, PT, and IRR criteria. The generalized quantitative PLR arises due to the uncertainty of these quantitative values, which are represented as fuzzy numbers with an arbitrary form membership function.

The algorithm consists of the following steps.

Step 1. Preparation of initial data.

This step aims to prepare the data needed to calculate the project's cash flows. This data includes the discount rate and partial data from which the incomes and expenses flows of the project will be calculated. Various methods (overview in Atra & Thomas, 2009) can be used to determine the discount rate. The composition of partial data depends on the specifics of a particular project. For example, if project incomes will be calculated based on the equilibrium point of the sales market, then the source data should describe a demand curve and a supply curve of the market. Or if the calculation of project expenses takes into account not only staff salaries, but also social costs, then at this step their components should be determined.

Step 2. Calculation of project cash flows.

The calculation of the project's cash flows involves the calculation of the values of partial criteria for the performance of the project for each time t. These criteria include the following:

project expenses $\widetilde{Ex_t}$;
project incomes $\widetilde{In_t}$;
discounted project expenses $\widetilde{Ex_t^D}$;
discounted project incomes $\widetilde{In_t^D}$;
discounted profit of the project $\widetilde{Pr_t^D}$.

These criteria are calculated as follows:

$$\widetilde{In}_t^D = \frac{\widetilde{In}_t}{\left(1+\widetilde{DR}_t\right)^t}, \widetilde{Ex}_t^D = \frac{\widetilde{Ex}_t}{\left(1+\widetilde{DR}_t\right)^t}, \quad (10)$$

$$\widetilde{Pr}_t^D = \widetilde{In}_t^D - \widetilde{Ex}_t^D, \ \forall\, t = \overline{1,T}$$

where $\widetilde{DR}_t$ – the value of the discount rate at some in time t;
T – project end time.

Project expenses are the sum of all project expenditure items: purchase and installation of equipment; salary; taxes; repayment of a credit; purchase of raw materials and others. Part of the costs may depend on the volume of products produced, but this is not important for our study.

The project incomes are the result of sales of project products on the market: $\widetilde{In}_t = \widetilde{Pr}_t \ \widetilde{Q}_t$, where $\widetilde{Pr}_t$ is the price of one unit of the project product, and $\widetilde{Q}_t$ is the number of sold units of project products. The income of the project can be determined in different ways, depending on the chosen strategy for promoting the product on the market. If the market volume is significant and the market or without problems absorbs all the products of the project, it is necessary to use the equilibrium price of the market, which is defined as the intersection of the supply and demand curves (Whelan et al., 2001). If management sets a goal to capture the market, it will sell the project's products with a reduced price, and then the project's incomes should be determined based only on the demand curve. There are other variants, the consideration of which is not the subject of our study.

Step 3. Calculation of project performance criteria.

The generalized numerical criteria of the project are the net present value of the project $\widetilde{NPV}$, the project payback period PP and the internal rate of return of the project $\widetilde{IRR}$. These criteria can also be calculated according to the standard scheme:

$$\widetilde{NPV} = \widetilde{Pr}_T^D, PP = \quad (11)$$

$$\min_{t=\overline{1,T}} t \mid \widetilde{NPV}^{MEV} > 0, \widetilde{IRR} = \widetilde{DR}_{PP} \big| \widetilde{NPV}^{MEV} = 0$$

In expression (11), instead of $\widetilde{NPV}^{MEV}$, other characteristics of fuzzy numbers can be used, in particular (3), (4) and (6).

Step 4. Calculation of partial criteria for quantitative PLRω_n.

Above, we suggested using the characteristics of fuzzy numbers (see expressions 5-8), since fuzzy numbers themselves are poorly suited for decision making and must be defuzzified. Therefore, as partial criteria for quantitative PLR, we propose to use the characteristics of those fuzzy numbers that describe the project performance criteria. As example, for NPV, characteristic (3) is a pessimistic risk estimate, characteristic (4) is an optimistic estimate, characteristic (5) is the most expected estimate, and characteristic (6) is an equilibrium estimate. The first two characteristics use a level that can be conditionally called the level of risk appetite. This level reflects the manager's view of risk assessment.

Table 4 contains partial criteria for quantitative PLR that we propose to use.

We emphasize that this is the most complete list of partial criteria for quantitative PLR. In practice, these criteria can be used, depending on the personal preferences of the lender's management, its experience or necessity. In addition, other partial criteria can also be defined.

Step 5. Calculation of the generalized quantitative PLR.

Since the set of partial criteria $Y = \{y_n, n = \overline{1,N}\}$ is quite large, it is advisable to generalize them into one criterion J^{quan} to facilitate decision making and make the decision transparent. To generalize, we will use an approach similar to the calculation of qualitative PLRs. In particular, the following steps will provide a generalization of partial criteria. For the case indicated in Table 4, N=11.

Step 5.1. Converting numerical estimates of criteria of quantitative PLR to qualitative estimates.

Suppose that the values of partial criteria ω_n will be considered by several managers who will convert them into qualitative estimates described by the membership function $\sigma(y_n|f_j)$:$y_n \to [0,1]$. To obtain an estimate of $\sigma(y_n|f_j)$, the manager f_j must correlate the calculated numerical value of each criterion ω_n with the Harrington "desirability" curve and select from Table 3

Table 4. Partial criteria for quantitative PLR

№	Partial criteria for quantitative PLR ω_n	Explanations for partial criteria
1	Pessimistic estimate of NPV: $\widetilde{NPV}^{\alpha\, min}$	The minimum possible value of NPV for established level of risk appetite. The lower the estimate, the higher the risk of shortfall in project profits.
2	Optimistic estimate of NPV: $\widetilde{NPV}^{\alpha\, max}$	The maximum possible value of NPV for established level of risk appetite. The higher the estimate, the lower the risk of shortfall in project profits.
3	Most expected estimate of NPV: $\widetilde{NPV}^{MEV}$	The most possible value of NPV. The estimate takes into account the most possible combination of project risks. The higher the score, the lower the risk of shortfall in project profits.
4	Equilibrium estimate of NPV: $\widetilde{NPV}^{CG}$	The value of NPV under the condition of balance of factors for increase and decrease of risks. The higher the estimate, the lower the risk of shortfall in project profits.
5	Relative range of possible change of NPV: $\left(\widetilde{NPV}^{\alpha\, max} - \widetilde{NPV}^{\alpha\, min}\right)/\widetilde{NPV}^{MEV}$	A relative estimate of the possible change in NPV for established level of risk appetite. The assessment characterizes the strength of the impact of risks on the profit of the project. The higher the estimate, the higher the uncertainty of the project conditions and the higher the risk of shortfall in project profits.
6	Risk of pessimistic estimate of NPV: $\rho\left(\widetilde{NPV}^{\alpha\, min}\right)$	The possibility that the NPV value will be greater than the pessimistic estimate. The higher the estimate, the lower the risk of shortfall in project profits.
7	Risk of optimistic estimate of NPV: $\rho\left(\widetilde{NPV}^{\alpha\, max}\right)$	The possibility that the NPV value will be greater than the optimistic estimate. The higher the estimate, the greater the risk of shortfall in project profits.
8	Pessimistic payback period of the project, calculated on the basis of a pessimistic estimate of NPV: $PP^{pes} = \min_{t=\overline{1,T}} t \mid \left(\widetilde{Pr}_t^D\right)^{\alpha\, min} > 0.$	It often happens that the most expected estimate of NPV has become positive, but the carrier of the fuzzy number that describes NPV is partially negative. In this case, this criterion will show the possibility of increasing the payback period of the project.
9	The optimistic payback period of the project, calculated on the basis of the most expected estimate of NPV: $PP^{opt} = \min_{t=\overline{1,T}} t \mid \left(\widetilde{Pr}_t^D\right)^{\alpha\, max} > 0.$	The payback period of the project, which can be expected in case of positive conditions for the project.
10	The pessimistic internal rate of return of the project, calculated on the basis of the pessimistic estimate of NPV: $IRR^{pes} = \widetilde{DR}_{PP^{pes}} \| NPV^{\alpha\, min} = 0$	The internal rate of return of the project, which can be expected in case of negative conditions for the project.
11	The optimistic internal rate of return of the project, calculated on the basis of the optimistic estimate of NPV: $IRR^{opt} = \widetilde{DR}_{PP^{opt}} \| NPV^{\alpha\, max} = 0$	The internal rate of return of the project, which can be expected in case of positive conditions for the project.

the linguistic value k that best matches the manager's opinion. Then the manager must choose a numerical estimate $\sigma(y_n\ |f_j)\in h_k$ from the appropriate interval.

Step 5.2. Aggregation of managers' estimates whit taking into account their competence.

After all managers have defined their estimates, it is necessary to calculate the group estimate of each criterion $\vartheta(y_n)$, aggregated with taking into account the competence of managers. If the competence of managers is represented as a fuzzy measure $g_F(\cdot)$:$2^F\rightarrow[0,1]$, then the Sugeno fuzzy integral from the membership function $\sigma(y_n\ |f_j)$ along the fuzzy measure $g_F(\cdot)$, by analogy with (8), calculates group estimate $\vartheta(y_n)$ in each criterion from the set $y_n\in Y$:

$$\vartheta(y_n) = (s)\int_F \sigma(y_n|f_j) \circ g_F(\cdot)$$

Step 5.3. Aggregation of group estimates of partial criteria of quantitative PLR, taking into account the importance of the criteria.

Sugeno fuzzy integral from the membership function $\vartheta(y_n)$ along the fuzzy measure of criteria importance $\varphi(\cdot)$:$2^N\rightarrow[0,1]$ makes it possible to calculate the estimate of the generalized quantitative criterion PLR:

$$J^{quan} = (s)\int_Y \vartheta(y_n) \circ \varphi(\cdot)$$

Thus, the algorithm for assessing a generalized quantitative PLR provides:

-calculation of the values of the project performance criteria, presented in the form of fuzzy numbers;

-calculation of partial criteria of quantitative PLR as characteristics of fuzzy numbers that describe these performance criteria;

-calculation of the generalized quantitative PLR to simplify the decision to lend to the project.

4.3. Assessing the risks of lending to a project for the production of corn syrup in Ukraine

As follows from the description of the algorithms presented above, the assessment of PLRs requires a large amount of data. A full description of these data, together with intermediate data, requires a large amount of text. Therefore, we present here only a generalized description of the data and focus on the most important details.

4.3.1. Brief description of the project

The project was implemented in 1998, when Ukraine began to emerge from the economic crisis. The project was aimed at organizing the production and marketing of corn syrup from corn. Corn syrup is an important raw material in the confectionery industry and is used to sweeten and prevent crystallization of confectionery. The borrower intended to carry out the project at one of its own agricultural enterprises. Since the borrower's own funds were not enough, he turned to the lender in order to replenish working capital, as well as finance business expansion and market development. At that time, the relevance of the project was determined by the beginning of the growth in the welfare of the population and, accordingly, the growth in the capacity of the confectionery market, the growth of the confectionery industry and the volume of corn production. However, the economic situation in the country and the industry remained difficult. The borrower planned to complete the project in 8 months. On the lender side, PLRs were assessed by one manager.

4.3.2. Assessment of generalized qualitative PLR

We determined the initial project criteria values and calculated the estimates of PLRs according to the criteria hierarchy shown in Fig. 3. Table 5 shows criteria estimates of qualitative PLR.

As we can see, the generalized qualitative PLR is 0.547. If we will use the analogy for the risk assessment model (Frei & Ruloff, 1988) proposed by Frei and Rullof, this risk can be characterized as increased.

4.3.3. Assessment of generalized quantitative PLR

In accordance with the dependencies (10) and (11), we calculated the estimates of performance criteria, the most expected values of which are shown in Table 6. In the calculations, the level of risk appetite is set to α=0.7. The discount rate is represented by a fuzzy number that describes the value of "about 2%" per month.

For calculations, we used the developed add-in "Fuzzy for Excel" for Microsoft Excel office software. This add-in can be

Table 5. Estimates in criteria of qualitative PLR

Generalized criteria	J										
Estimation	0.547										
Intermediate criteria	x_1	x_2	x_3	x_4	x_5	x_6					
Estimations	0.538	0.556	0.361	0.455	0.761	0.547					
Intermediate criteria	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}				
Estimations	0.497	0.052	0.118	0.848	0.567	0.539	0.299				
Intermediate criteria	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}						
Estimations	0.73	0.146	0.254	0.407	0.556						
Intermediate criteria	x_{31}	x_{32}	x_{33}								
Estimations	0.403	0.024	0.024								
Intermediate criteria	x_{41}	x_{42}	x_{43}								
Estimations	0.209	0.316	0.455								
Intermediate criteria	x_{51}	x_{52}									
Estimations	0.761	0.455									
Intermediate criteria	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}	x_{67}	x_{68}	x_{69}	x_{610}	x_{611}
Estimations	0.216	0.674	0.299	0.205	0.101	0.295	0.547	0.299	0.278	0.209	0.466
Intermediate criteria	x_{17-1}	x_{17-2}	x_{17-3}	x_{17-4}							
Estimations	0.817	0.156	0.101	0.299							

Table 6. Estimates of performance criteria of the project, thousand UAH

Criteria	***t***							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
$\widetilde{In}_t$	0	0	1766	3640	5520	7870	9920	11990
$\widetilde{Ex}_t$	270	1025	2380	4000	5750	7477	9254.5	11000
$\widetilde{In}_t^D$	0	0	1664	3362	4999	6988	8623	10233
$\widetilde{Ex}_t^D$	270	985	2242	3695	5207	6639	8056	9388
$\widetilde{Pr}_t^D$	-270	-1028	-578	-332	-208	348	566	844

downloaded from the link (Fuzzy for Excel, 2022).

Figure 5 shows a fuzzy number that describes NPV, and Figure 6 shows the risk-function of this fuzzy number.

Table 7 shows: estimates of partial criteria for quantitative PLR; importance of criteria and the manager estimates converted with the help of a Harrington desirability curve.

As we can see, the project becomes profitable after 5 months of implementation. We expected NPV to be approximately UAH 850,000 by the end of the project. At a risk appetite level of 0.7, NPV will be at least UAH 800,000. The generalized quantitative PLR can be defined as tolerable with approach to the area of increased risk.

In general, despite the duality of these assessments, the lender decided to lend to the project, since a short payback period weakened the effect of some qualitative risks. In practice, the project was successfully implemented in 4 months against the backdrop of growing market capacity.

5. DISCUSSION

As we can see, the assessment of PLRs is a complex and time-consuming process. Let us further consider several important

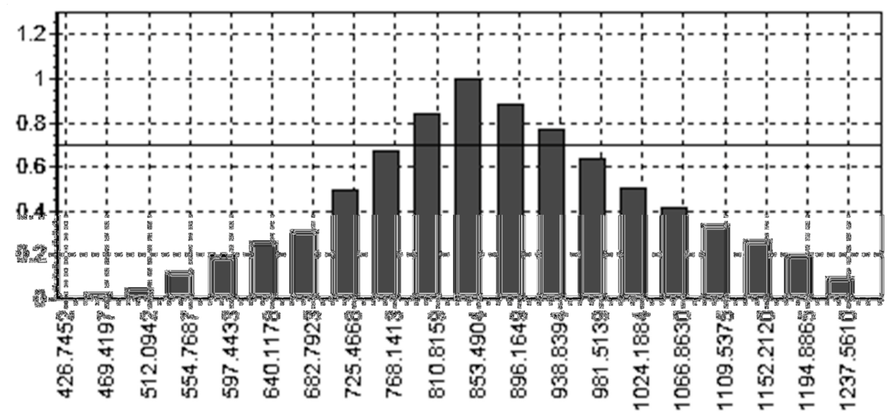

Figure 5. $\widetilde{NPV}$ *of the project*

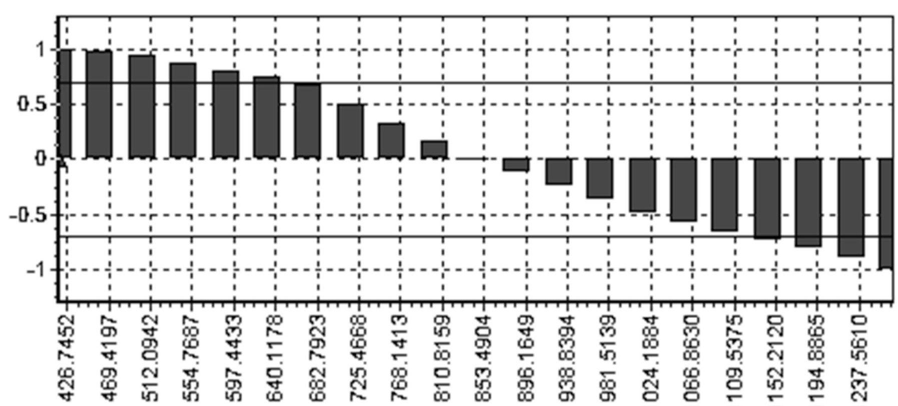

Figure 6. Risk-function of $\widetilde{NPV}$

Table 7. Estimates of partial criteria for quantitative PLR; importance of criteria and the manager estimates

№	Partial criteria for quantitative PLR	Estimates of partial criteria	Manager estimates $\vartheta(y_n)$	Criteria importance $\varphi(y_n)$
1	Pessimistic estimate of NPV: $\widetilde{NPV}^{\alpha\, min}$	802 000 UAH	0.463	0.313
2	Optimistic estimate of NPV: $\widetilde{NPV}^{\alpha\, max}$	930 000 UAH	0.183	0.062
3	Most expected estimate of NPV: $\widetilde{NPV}^{MEV}$	844 000 UAH	0.366	0.09
4	Equilibrium estimate of NPV: $\widetilde{NPV}^{CG}$	876 000 UAH	0.327	0.144
5	Relative range of possible change of NPV: $\left(\widetilde{NPV}^{\alpha\, max} - \widetilde{NPV}^{\alpha\, min}\right)/\widetilde{NPV}^{MEV}$	15%	0.366	0.19
6	Risk of pessimistic estimate of NPV: $\rho\left(\widetilde{NPV}^{\alpha\, min}\right)$	+0.163 (stability area)	0.533	0.157
7	Risk of optimistic estimate of NPV: $\rho\left(\widetilde{NPV}^{\alpha\, max}\right)$	-0.234 (risk area)	0.475	0.09
8	Pessimistic payback period of the project, calculated on the basis of a pessimistic estimate of NPV: $PP^{pes} = \min_{t=\overline{1,T}} t \mid \left(\widetilde{Pr}_t^D\right)^{\alpha\, min} > 0.$	5 months	0.101	0.228
9	The optimistic payback period of the project, calculated on the basis of the most expected estimate of NPV: $PP^{opt} = \min_{t=\overline{1,T}} t \mid \left(\widetilde{Pr}_t^D\right)^{\alpha\, max} > 0.$	5 months	0.093	0.123
10	The pessimistic internal rate of return of the project, calculated on the basis of the pessimistic estimate of NPV: $IRR^{pes} = \widetilde{DR}_{PP^{pes}} \mid NPV^{\alpha\, min} = 0$	80%	0.506	0.062
11	The optimistic internal rate of return of the project, calculated on the basis of the optimistic estimate of NPV: $IRR^{opt} = \widetilde{DR}_{PP^{opt}} \mid NPV^{\alpha\, max} = 0$	150%	0.148	0.038
Generalized quantitative PLR				0.463

questions that relate to the features of the proposed algorithms.

5.1. The question about using Sugeno fuzzy integral to generalize partial criteria.

As we said earlier, the logic of generalizing risks using the Sugeno fuzzy integral depends on the λ parameter of the fuzzy measure of the importance of partial risks $g_A(\cdot)$. Consider the top level of criteria in the hierarchy (see Fig. 3). Using expression (9), we calculate the value of the generalized criterion J^{qual} for several values of the λ parameter $g_A(\cdot)$, in particular: λ=-1, λ=0 and $\lambda\approx$10. For these cases, we built fuzzy measures $g_A(x_i)_{\lambda=-1}$, $g_A(x_i)_{\lambda=0}$ and $g_A(x_i)_{\lambda\approx 10}$ based on the measure $g_A(\cdot)$ from Table 1, keeping the proportions between the densities of the fuzzy measure. In expression (9), we used the criteria estimates from Table 5. The calculation results are presented in Table 8.

As the calculation results show, the use of the minority logic ensures the choice of the maximum estimate from all partial estimates. This logic formalizes the lack of risk appetite and the maximum caution of the lender. Using the logic of the majority reduces the risk estimate. This logic formalizes the case of high risk appetite.

Using the weighted average method provides an estimate similar to the initial variant. However, the weighted average method has two well-known serious drawbacks. First, it is the insensitivity of the generalized estimation in the case of a large number of partial criteria, when most of the estimates compensate even significant changes in one estimate. In practice, this property begins to manifest itself already at 5-6 partial criteria. And secondly, this is a significant complication of the aggregation procedure if the criteria depend on each other. As follows from the research of Saaty & Kearns (1985), taking into account the correlations of criteria is a very laborious procedure.

Thus, the use of Sugeno fuzzy measures with different modalities and preservation of preferences between partial criteria provides the decision-making process with additional data. In addition, when the risk appetite changes, it is enough to change the normalization parameter of fuzzy measures that describe the importance of partial criteria without rebuilding the structure of preferences.

5.2. The question about developing recommendations to reduce generalized qualitative PLR

The lender is not always limited to assessing the risks of the project and deciding regarding its lending or refusal to lend. It often happens that in practice the risks of the project are high, but approaching to the level of a positive decision. In this case, the consequence of refusing to lend may be loss of profit. Therefore, in order to reduce risks, it is advisable to develop recommendations for adjusting some project options and/or a loan request.

In our case, the development of recommendations involves the definition of such initial criteria (see Figure 3), the change in the estimates of which ensures the maximum reduction in the generalized qualitative PLR. To solve this problem, we can use the property of the Sugeno fuzzy integral calculation procedure, which follows from expression (1). If the membership function *h(x)* is sorted in descending order, then the Sugeno integral calculation technique can be illustrated as shown in Figure 7.

As can be seen from Figure 7, the integration result takes into account only the criteria x_i' from the subset H, which are cut off by the intersection point of the decreasing membership function and the increasing fuzzy measure. Only these criteria influenced the result of integration. A change in these criteria will ensure a change in estimate of the generalized risk.

Therefore, if the identification of elements x_i' starts from the top level of the hierarchy (see Figure 3) and continues going

Table 8. Dependence of the result of integration on the λ parameter of the fuzzy measure of importance $g_A(\cdot)$

Intermediate criteria	x_1	x_2	x_3	x_4	x_5	x_6	J
Estimates	0.538	0.556	0.361	0.455	0.761	0.547	
$g_A(x_i)_{\lambda=-1}$	0.376	0.5	0.29	0.667	0.667	1	0.667
$g_A(x_i)_{\lambda=0}$	0.109	0.143	0.085	0.188	0.188	0.287	0.547
$g_A(x_i)_{\lambda\approx 10}$	0.025	0.039	0.017	0.062	0.062	0.117	0.495

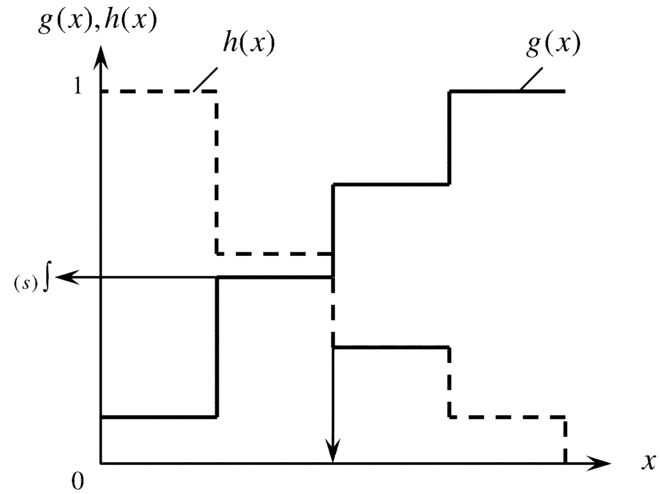

Figure 7. Technique for calculation of Sugeno fuzzy integral

down the hierarchy, then as a result we will be able to determine the list of initial project criteria that influenced the estimate of the generalized PLR. These criteria can be called "critical" project parameters that need to be changed in order to reduce the generalized PLR.

5.3. The question about using the risk-function

From the view-point of risk ontology, the discussed above characteristics $\widetilde{NPV}^{\alpha\, min}$, $\widetilde{NPV}^{\alpha\, max}$, $\widetilde{NPV}^{MEV}$ and $\widetilde{NPV}^{CG}$ describe the size of expected losses, which is not complete description of the risk. Although we can to identify these magnitudes with risks, nevertheless, the risk requires additional description in terms of the occurrence of losses in terms of relation to reality or in terms of modality (see above). For example, we say: in the current conditions, the creditor can lose $1.5 million. This is the amount of possible losses. But the question remains, to what extent this estimate can be implemented in practice? In other words, since we are talking about assessing losses in the future, risk assessment should include not only the amount of losses, but also an estimate of the occurrence (appearance) of this event.

In turn, three additional questions appear here. First, the question about modality as a relation to reality. Second, the question about describing risk appetite when the quantitative assessing. And thirdly, the question about gains as a category inverse to the category of risk.

5.4. The question about modality as a relation to reality

Since any fuzzy number is represented as a normal membership function (2) defined on a discretized set of real numbers, then such a fuzzy number is a fuzzy measure of possibility, the normalization parameter of which is λ=-1. Risk-function of the fuzzy number (7) fully complies with this statement, since it is calculated as a possibility. Thus, the risk-function proposed by us can be used to describe not only the size of the expected losses, but also the possibility of realizing these losses.

5.5. The question about describing risk appetite when the quantitative assessing

However, we argued above that when assessing qualitative risks, modality can be used to formalize the lender's risk appetite. The question arises whether it is possible to describe risk appetite in a similar way when assessing quantitative risks, if a fuzzy number has a single modality - possibility?

Such an analogy is the α level of confidence, which determines the estimates of partial criteria of quantitative risk: $\widetilde{NPV}^{\alpha\, min}$, $\widetilde{NPV}^{\alpha\, max}$, $(\widetilde{NPV}^{\alpha\, max} - \widetilde{NPV}^{\alpha\, min})/\widetilde{NPV}^{MEV}$ and others (see Table 3). This analogy is quite acceptable, since the set $H_i = \{x_i \mid h(x_i) \geq h(x_i),\ j = \overline{1,N}\}$ from expression (1) can be represented as a α-level set if α=$h(x_i)$. According to this, if $\lambda \to \infty$ (necessity), more elements x_j will fall into the set H_α, that is, more risk criteria will be considered in a result of integration (1). This will correspond to a low risk appetite. Accordingly, λ=-1 will describe a high-risk appetite, since only criteria with the highest estimates will be considered in a result of integration.

Similarly, an increase in the α level reduces the number of considered risk factors (increases risk appetite), and a decrease in the α-level, on the contrary, is equivalent to a decrease in risk appetite. Thus, our proposal to describe risk appetite using a λ-parameter (for qualitative risks) and a α-level (for quantitative risks) is non-contradictory.

5.6. The question about gains as a category inverse to the category of risk

Based on the international definition of risk (Purdy, 2010), uncertainty factors can be identified with risk factors. In the domain of the fuzzy number, there is always the most expected value, which divides the fuzzy number into two parts. If this fuzzy number describes a project performance criterion which it is desirable to increase (for example, profit), then the following reasoning is true. Values that are less than the most expected value are the result of risk factors, since they indicate a possible reduction in the criterion estimate. Values that are greater than the most expected value are the result of anti-risk factors, as they indicate the possibility of increasing the criterion estimate. Thus, the area of definition of a fuzzy number can be divided into two areas: the area of risk and the area of anti-risk (the area of stability). If the fuzzy number describes a criterion that it is desirable to reduce, then the areas of risk and stability are reversed. Proposed by us risk-function of the fuzzy number can be used to describe not only the risks of losses, but also the possibility of gains.

6. CONCLUSIONS

In this study, we proposed to jointly assess both qualitative and quantitative PLRs, since some of the risks are determined by qualitative, and some - by quantitative factors. The use of estimates of both qualitative and quantitative PLRs provides the most comprehensive data for lender decision-making regarding project lending.

To assess qualitative risks, we proposed a hierarchical structure of criteria that describe qualitative magnitudes. To represent criteria estimates, we used a fuzzy membership function defined on a discrete set of criteria. Sugeno fuzzy measure was used to describe the importance of the criteria. We used Sugeno fuzzy integral to generalize criteria estimates based on their importance. The use of a Sugeno fuzzy measure and the Sugeno fuzzy integral provides the possibility of implementing different aggregation logic in different fragments of the criteria hierarchy. Another advantage of using the Sugeno fuzzy integral is the ability to determine the initial parameters of the project and / or loan request, the change of which reduces the generalized qualitative PLR. In addition, the fuzzy measure normalization parameter can be used to describe the lender's appetite for qualitative PLRs.

Based on the Zadeh extension principle, to assess quantitative risks, we proposed using standard characteristics of fuzzy numbers that describe standard project performance criteria: net present value, payback period, and internal rate of return. In addition, we proposed a new risk characteristic – a risk-function of a fuzzy number, which describes not only the size of possible losses, but also the possibility of their occurrence. The level of confidence that is used to calculate the characteristics of fuzzy numbers can serve as a description of the lender's appetite for quantitative PLRs. To calculate the project performance criteria, we used standard arithmetic procedures that calculate cash flows. To perform arithmetic operations, we proposed to use the well-known fuzzy arithmetic algorithm based on the maximum entropy principle.

To calculate the generalized estimates of qualitative and quantitative PLR, we have developed appropriate algorithms that process the initial estimates of several experts. These algorithms consider the experts' competence, which is described using Sugeno fuzzy measure. We used the Harington "desirability" curve to obtain initial estimates.

In our opinion, it is advisable to use the developed algorithms to assess the risks of lending to small-scale production projects, since lending to large production projects has its own features and requires the use of other criteria, such as political support. In addition, the assessment of the risk of the loan portfolio is not considered here. This procedure should be considered after the risk assessment of each project, but this is a topic that requires a separate analysis. However, this does not negate the conclusions regarding the effects of using fuzzy integral calculus and fuzzy numbers. We consider the development of scoring procedures for the evaluation of personal credit risk as a possible direction for further research.

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АЛГОРИТМИ ЗА ПРОЦЕНУ КВАЛИТАТИВНИХ И КВАНТИТАТИВНИХ РИЗИКА КРЕДИТИРАЊА МАЛИХ И СРЕДЊИХ ПРЕДУЗЕЋА ЗАСНОВАНИ НА ФАЗИ РАЧУНУ

Sergey Sveshnikov, Victor Bocharnikov, Tatjana Uvarova, Petr Kovalchuk

Извод

Данас кризни услови у привреди и финансијама захтевају квалитетну процену ризика. У чланку аутори предлажу два алгоритма за процену ризика кредитирања пројеката (РКП) за мала и средња предузећа. За процену квалитативних РКП-а, предложили смо употребу хијерархијског система критеријума, у коме се важност критеријума описује коришћењем Сугено фази мере, а генерализована процена квалитативног ризика се израчунава коришћењем Сугено фази интеграла. Да бисмо проценили квантитативне РКП, предложили смо да се користе карактеристике фази бројева који описују критеријуме ефективности пројекта и имају произвољни облик функције припадности. Поред тога, за описивање квантитативних ризика, предложили смо да се користи функција ризика фази броја, која одражава не само величину могућих губитака, већ и могућност њиховог настанка. Ово вам омогућава да свеобухватно и објективно процените ниво ризика. Ове алгоритме смо демонстрирали и дискутовали на примеру припреме података за доношење одлуке о кредитирању пројекта за производњу кукурузног сирупа у Украјини.

Кључне речи: ризик, процена, фази мере, фази интеграл, фази број

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