www.sjm06.com

Serbian Journal of Management 4 (2) (2009) 227 - 238

Serbian Journal of Management

APPLICATION OF LINEAR PROGRAMMING IN ENERGY MANAGEMENT

Snežana Dragićević*[a] and Milorad Bojić[b]

[a] *University of Kragujevac, Technical faculty Čačak , Serbia*
[b] *University of Kragujevac, Faculty of Mechanical Engineering Kragujevac, Serbia*

(Received 12 May 2009; accepted 18 October 2009)

Abstract

As energy and equipment costs increase, efficient energy systems become more important in the overall economics of process plants. This paper presents a method for modeling and optimizing an industrial steam-condensing system by using linear programming (LP) techniques. A linear programming method is used to minimize the total costs for energy used net costs in steam-condensing systems. The LP technique will determine optimum values for the process design variables, so as to achieve minimum cost.

Keywords: Linear programming, Energy management, Minimum operating costs.

1. INTRODUCTION

Energy systems are usually complex systems within which elements and subsystems are highly correlated, and their behavior is hardly predictable without application of exact mathematical methods. Under the conditions of limited energy sources, the optimal economic effects of an energy process can be achieved only if optimal relationships of all components of the system are kept permanent. In addition, under the conditions of increased prices of energy systems, optimal operation of energy systems tend to be very relevant.

A number of optimization methods are available and these are subject to the very nature of a problem, the level of depth of the analysis. One of the simplest method for determination of the optimal solution in problems with several alternative solutions is the linear programming. The LP procedure will maximize or minimize a linear objective function which consists of unknown problem

* *Corresponding author: vedra.md@nadlanu.com, snezad@tfc.kg.ac.rs*

variables and constant coefficients. The variables must be constrained by equality or inequality relationships. It is very important that mathematical model should be a proper presentation of the problem, because incorrect statements make incorrect results.

For the analysis of engineering problems there are usually a great number of data. Computers enable quickly solutions regardless of complexity of the problem. Today optimization is inconceivable without the usage of computers.

In stead of experimental analyses in practice it is possible to realize experimental analyses on mathematical model due to linear programming technique. When satisfactory results are achieved then they can be applied in practice.

The preview of literature shows that the LP method has been frequently applied in energy-engineering optimizations: in process chemical industry where total annual cost was minimized (Grossman & Santibanez, 1980), for numerical optimization technique for engineering design with applications (Vanderplaats, 1984), in system with a gas turbines and heat pumps where operational costs was minimized (Spakovsky et al., 1995), for analyzed minimal cost of supplying in building with heat of residential heating system (Gustafsson & Bojić, 1997) for minimal operating expenses for energy in the CHP energy system (Bojić & Stojanović, 1998), in the energy system with a condensing steam turbine (Bojić et al., 1998) and for thermal storage system of non-industrial utilities where daily operational cost was minimized (Yokohama & Ito, 2000).

In this paper we have studied energy system, which consist of a power plant that generated energy and the technology that consumed energy. The power plant consisted of a steam boiler, a turbine which was with steam extraction, and a heat pump (Fig. 1). The steam boiler generated high pressure steam, the turbine generated electricity and extracted steam whereas the heat pump generated low pressure steam. Steam and electrical energy could also be taken from outside sources. The technology consumed heat and electricity, and the heat pump consumed electricity and low grade steam.

The objective of this study was to identify energy sources which, for different unit costs for energy, minimized the total costs for energy used in the technology. In this study, the unit prices of heat and power produced outside the factory was assumed fixed, i.e. regulated by the government. The unit costs for heat and power produced inside the system were assumed to be varying, i.e. system management could control them in order to minimize them.

2. LP APPLIED TO THE STEAM-CONDENSING SYSTEMS

2.1. Linear programming – a brief review

Linear programming is one of several mathematical technique that attempt to solve problems by minimizing or maximizing a function of several independent variables. LP is widely used of these methods and is one of the best for analyzing complex industrial systems. The LP technique is more flexible than methods based on solving a system of equations for heat and material balances. For a desired set of conditions with a specified objective, a final solution is obtained in a single computer run – thus eliminating the numerous solutions required by the case study approach. When nonlinear variables are introduced in problem LP solutions becomes impossible due to the nonlinear nature of the energy balance equations. The iterative technique gets around this difficulty by solving successive linear approximations of the nonlinear equations. With each new solution, the linear approximations are improved. The final solution is achieved when all approximations are within a small tolerance of the actual nonlinear equations.

General mathematical formulation of linear programming procedure comprises the following: finding a group of variables x_1, x_2, ..., x_n which satisfy system of linear equation or inequation:

$$\sum_{k=1}^{n} a_{ik} \cdot x_k + b_i \geq 0, \quad i = 1,2,...,m$$
$$x_k \geq 0, \quad k = 1,2,...,n \tag{1}$$

where a_{ik} are constant coefficients, the x_k are unknown problem variables, the b_i are constant coefficients, and the m are number of constraints. The LP procedure will maximize or minimize a linear objective function of the form:

$$F(x_1, x_2,..., x_n) = c_1 \cdot x_1 + c_2 \cdot x_2 + ... + c_k \cdot x_k + ... + c_n \cdot x_n \tag{2}$$

where the c_k are constant coefficients. All LP solution codes require that all variables be nonnegative $x_k \geq 0$.

Practical engineering analyses may contain nonlinear terms. The non-linear programming optimization method is more complicate than linear programming method. To apply the LP technique all non-linear terms must be linear. The non-linear terms have to be linearized by using Taylor series expansions:

$$\begin{aligned} F(x_o + \Delta x, y_o + \Delta y, ...) &= F(x_o, y_o, ...) + \\ &+ \frac{1}{1!}\left(\frac{\partial}{\partial x}\Delta x + \frac{\partial}{\partial y}\Delta y + ...\right) \cdot F(x_o, y_o, ...) + \\ &+ \frac{1}{2!}\left(\frac{\partial}{\partial x}\Delta x + \frac{\partial}{\partial y}\Delta y + ...\right)^2 \cdot F(x_o, y_o, ...) + ... \\ \Delta x = x - x_o, &\quad \Delta y = y - y_o \end{aligned} \tag{3}$$

In most cases, the nonlinear product must be replaced by a first-order Taylor-series expansion. For example:

$$\begin{aligned} F(x, y) &= x_o \cdot y_o + (x - x_o) \cdot y_o \\ &+ (y - y_o) \cdot x_o = \\ &= x_o \cdot y + y_o \cdot x - x_o \cdot y_o \end{aligned} \tag{4}$$

Limitation for the usage of the linear programming method is degree of nonlinearity of the problem. The higher degree of nonlinearity of the system the lower potentional for the usage of linear programming method. The efficiency of usage of the linear programming method for optimization of energy systems depends on specific problem. The linear programming usage procedures on steam-condensing systems are defined by (Clark & Helmick, 1982).

2.2. Modeling the steam-condensing system

In this paper we have analized energy system shown in Fig. 1. For selection of energy inputs, an optimization LP method was applied. The technology power needs

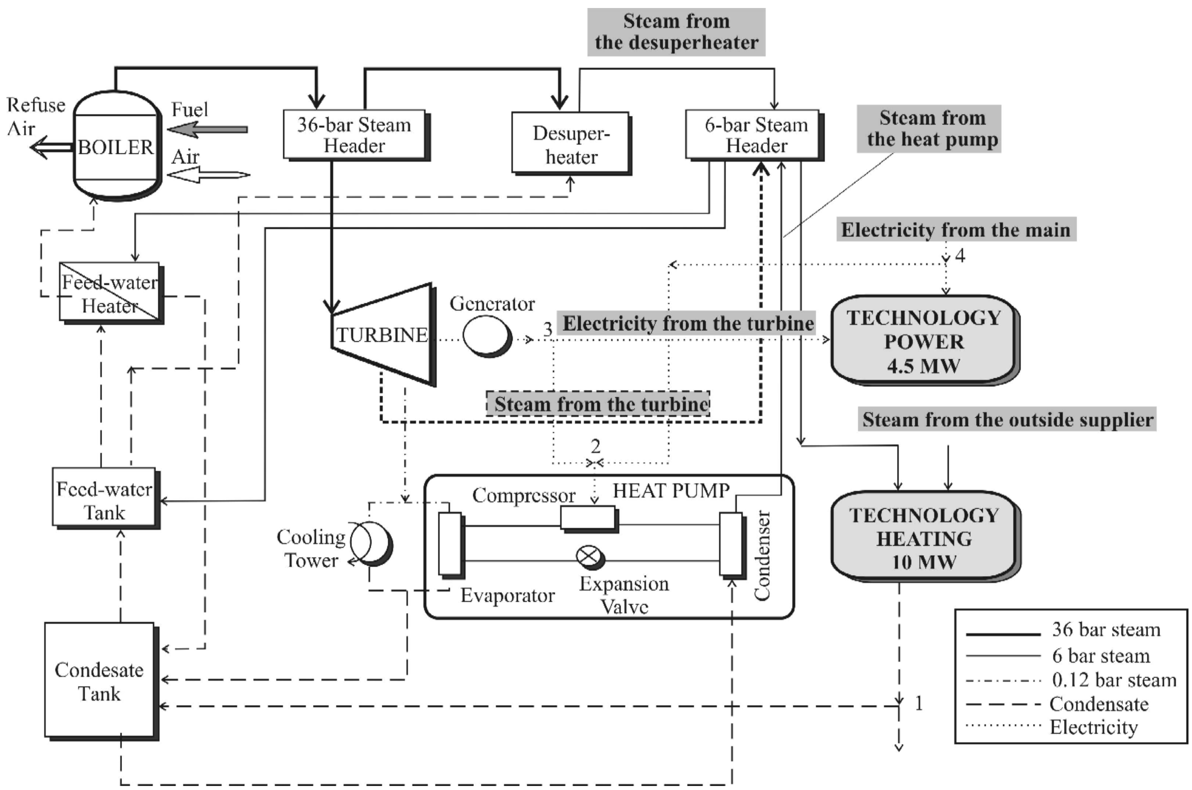

Figure 1. Schematic of the energy module network of the steam-condensing system

4.5 MW electricity, and the technology heating needs 10 MW steam heat.

This power plant might operate by using the following devices: (1) the boiler, turbine and desuperheater when the desuperheater generated steam and the turbine generated electricity and extracted steam, (2) the boiler, turbine and heat pump when the heat pump generated the steam and the turbine generated electricity and extracted steam, and (3) the boiler and desuperheater when the desuperheater generated the steam.

The technology might consume four types of the steam: (1) steam obtained from the desuperheater, (2) steam extracted from the turbine, (3) steam generated at the heat pump, and (4) steam generated outside the factory. The technology might consume two types of electricity: (1) electricity generated at the turbine and (2) electricity taken from the line grid. The heat pump consumed steam from the turbine and might consume two types of electricity: (1) electricity generated at the turbine and (2) electricity taken from the line grid.

The first task is to define a mathematical model representing the analyzed system. The bottom-up method was used to develop the energy module network for this energy system, shown in Fig. 1. This is accomplished by using input data modules for each for the process units in the system. Table 1 shows the units for which data modules have been developed. Each module is defined by completing a set of “fill in the blank” data sheets. Specific inputs for a given unit module include operating data required to define the unit's performance. This information is used to generate the constraint and energy and material balance equations for the unit. These modules were connected with the flow paths of steam, condensate and electricity. Based on this network, the non-linear equation system, having 26 equations was derived. Thirteenth of these equations were non-linear and 13 were linear.

Table 1.Modular LP models for the steam-condensing system from Fig. 1

1.	Steam boiler	variable flow, fixed enthalpy
2.	Steam headers	variable flows, fixed enthalpy
3.	Desuperheater	variable flow, fixed enthalpy
4.	Turbine	variable power demand
5.	Cooling tower	variable flow, fixed enthalpy
6.	Technology power	fix power demand
7.	Technology heating	fix power demand
8.	Heat pump	variable demand
9.	Condensate tank	variable flows, fixed enthalpy
10.	Feed water tank	variable flows, fixed enthalpy
11.	Feed water heater	variable flow, fixed enthalpy

The set of equations that describes the analyzed steam-condensing system consists of the following relationships:

1. Equations for the material balances for each process unit and equipment item:

$$\sum_{i=1}^{n} m_{i,out} - \sum_{k=1}^{m} m_{k,in} = 0 \qquad (5)$$

where the $m_{i,out}$ is output mass flows and $m_{i,in}$ is input mass flows.

2. Equations for the energy balances for each process unit and equipment item:

$$\sum_{i=1}^{n} (h_i \cdot m_i)_{out} - \sum_{k=1}^{m} (h_k \cdot m_k)_{in} = 0 \qquad (6)$$

where the $(x_i \cdot m_i)_{out}$ are output mass flows and enthalpy variables and the $(x_k \cdot m_k)_{in}$ are input mass flows and enthalpy variables.

3. An equations for each steam power Q and electric power E demand:

$$\sum_{i=1}^{n} Q_{i,out} - \sum_{k=1}^{m} Q_{k,in} = 0 \qquad (7)$$

$$\sum_{i=1}^{n} E_{i,out} - \sum_{k=1}^{m} E_{k,in} = 0 \qquad (8)$$

4. Upper and lower bounds for all independent and equalities and upper/lower bounds that are problem depended. This mathematical model had constraints, which specified the plant design:

$$m_{i,min} \le m_i \le m_{i,max}\,,\ f \le 1\,, g \le 1\,, d \le 1 \qquad (9)$$

where:

- $m_{i,min}$ and $m_{i,max}$ are minimum and maximum mass-flow rate of the steam at the entrance of the turbine and minimum and maximum amount of saturated steam at the exit of the turbine;
- f presents proportions the total amount of steam that would be cooled in the cooling tower and that would be cooled in the heat pump;
- g presents proportions the total amount

of turbine electricity which would be consumed by the heat pump and which would be consumed in the technology;

- d presents proportions the total amount of main electricity which would be used by the heat pump and which would be used in the technology.

2.3. The objective function

The objective function for a steam system can range from very simple to quite complex. The simplest consists of a single variable. For example, it represents either the steam-generation rate, or the total fuel consumed. In either case, the objective function is minimized. An alternative objective function minimizes total operating expense. It includes costs for fuel, boiler feed water makeup, electric power, cooling water makeup, and catalysts and chemicals for water treating. This type of objective function should be used by operating companies when the goal is minimum operating expense for an existing system.

A more comprehensive objective can be defined by including operating expense, and the cost of capital recovery plus return on investment for major equipment. For a new steam system in the design stage, this objective function represents the total variable cost of the system and is minimized. Selection of a particular objective function depends on the purpose of the study.

In this paper, the linear objective function was economic in nature and involved cost minimizations. This cost function depended on the amounts of the different energy inputs and their unit costs:

$$F = \sum_{i=1}^{4} C_i \cdot Q_i + \sum_{k=1}^{4} C_k \cdot E_k \tag{10}$$

where the C_i are unit costs for steam power and the C_k are unit cost for electricity. The objective was to minimize the operating expenses in the factory that consumed heat and electricity. The steam power demand can be satisfied by either from steam generated by the factory power plant by using its desuperheater, its turbine with steam extraction and its heat pump, or produced by outside supplier. The electric power demand can be satisfied by either the turbine, or taken from the main. The turbine electrical energy could be used by the technology, and by the heat pump. The line grid electrical energy also could be used by the technology, and by the heat pump.

2.4. The nonlinear problem and linearization

The relationships given by Eq. (6) may contain nonlinear terms such as m·h, where (m) represents an unknown mass flow rate, and (h) an unknown enthalpy. The constraint relationships must be linear because the simplex algorithm, used by the LP solution codes, cannot handle nonlinear terms. To get around this problem, it is possible to replace the nonlinear terms with single variables, or with linear approximations, in order to form a linear model that can be solved by using LP methods. The approximations require estimates of coefficients. When the estimates are correct, the errors introduced by the approximations become insignificant. In this investigation iterative estimation of coefficients is used, and subsequent problem solution, until a stable set of coefficients is found that satisfies maximum error requirements. The LP solution that utilizes this last set of coefficients is the solution that we are seeking.

In most cases, the nonlinear product m·h must be replaced by a first-order Taylor-series expansion. For example:

$$m \cdot h \approx m_o \cdot h + m \cdot h_o - m_o \cdot h_o \quad (11)$$

In Eq. (11) m_0 and h_0 are Taylor-expansion coefficients, and are the best available estimates of the true values of m and h. Values for the coefficients must be estimates initially, and then reevaluated by successive LP solutions until a tolerance test can be met.

A similar linearization technique is used when nonlinear terms are encountered in the objective function. This usually occurs when the capital costs of equipment items are represented as function of problems variables. In this paper the objective function is linear.

2.5. Solution technique

A number of computer codes have been developed to solve LP models. Many of these codes are available today. In this study the operating costs were optimized, i.e. minimized in an iteration procedure by using the previously developed equalities and inequalities and the physical and operational parameters of the system. To apply LP technique, we used 26 linear equations, the objective function and constraints. The values for the linearized variables were compared to their assumed values. If the calculated values differed from the input values, the calculated values were taken as the initial values for the new iteration. The iteration procedure was repeated until all the calculated values were almost equal to the input values.

3. RESULTS

The optimisation yielded five scenarios of energy use in the factory, which are shown in Fig. 2 by using five energy-mix regions. Each energy-mix region presents combinations of unit cost of steam generated at the desuperheater and unit cost of electricity generated by the turbine, which requires a specific energy mix to be used in the technology, giving the smallest costs for energy. The unit cost for grid electric was C_m= 0.25 €/kWh, and the unit cost for heat generated by steam from external source was C_{so}= 0.1 €/kWh.

For each particular energy-mix region in Fig. 2 and Fig. 3 presents optimum mix of steam inputs, and Fig. 4 presents the optimum mix of electricity inputs. Each energy-mix region represents one optimal energy-mix scenario.

Region 1: For region 1 of Fig. 2, the smallest cost was obtained when the turbine simultaneously generated the steam (see Fig. 3) and electricity (see Fig. 4). The operating costs in this region were recorded up to 2125 €/h. The lowest cost 0 €/h would be matched when refuse energy (free fuel) was available to be used in the factory boiler. In this case, the boiler and the turbine operated and the heat pump and the desuperheater did not operate.

Region 2: In region 2 of Fig. 2, the electricity and the steam was produced in the factory. Fig. 3 shows that the heat Q_{6ex} = 8250 kW was generated by the turbine, and the heat of Q_{hp} = 1750 kW was generated by using the heat pump. Fig. 4 shows that the electricity of E_t =5200 kW was used in the factory, where E_{tt} = 4500 kW would cover the technology, and E_{tc} =E_c = 700 kW would cover the heat pump. Therefore, in this region, the boiler, the turbine and the heat pump operated and the desuperheater did not operate.

Region 3: In region 3 of Fig. 2, the whole amount of the steam was produced in the factory, because the unit cost for the steam which generated in the factory was lower than that might be purchased from the line grid. In region 3 of Fig. 2, two boundary optimum electric-energy mixes were found to exist. These mixes were denoted 3(1) and 3 (2) in Fig. 4. Both electric-energy mixes used the minimal amount of boiler steam to run the turbine and generated the electricity, and remainder of electricity was purchased from the line grid. In region 3 it was possible to use all electric-energy mixes, which were among two boundary presented energy mix 3(1) and 3(2). The operating costs recorded in region 3 were in the range from 867 to 2125 €/h.

Region 4: In region 4 of Fig. 2, the steam was produced at the desuperheater (see Fig. 3), and electricity was purchased from the line grid (see Fig. 4). The unit cost for the steam produced at the desuperheater was lower than that of the steam that might be purchased from the outside source or produced using by the heat pump or by the turbine. The unit cost for electricity purchased from the line grid was lower that that might be produced in the factory. In this case, the boiler and the desuperheater operated, and the turbine and the heat pump did not operate. The operating costs in this region were recorded in the range from 1125 to 2125 €/h. The lowest cost of 1125 €/h would be matched when refuse energy was available to be used in the boiler. Two selected intermediate values of the optimum costs are given in region 4 of Fig. 2.

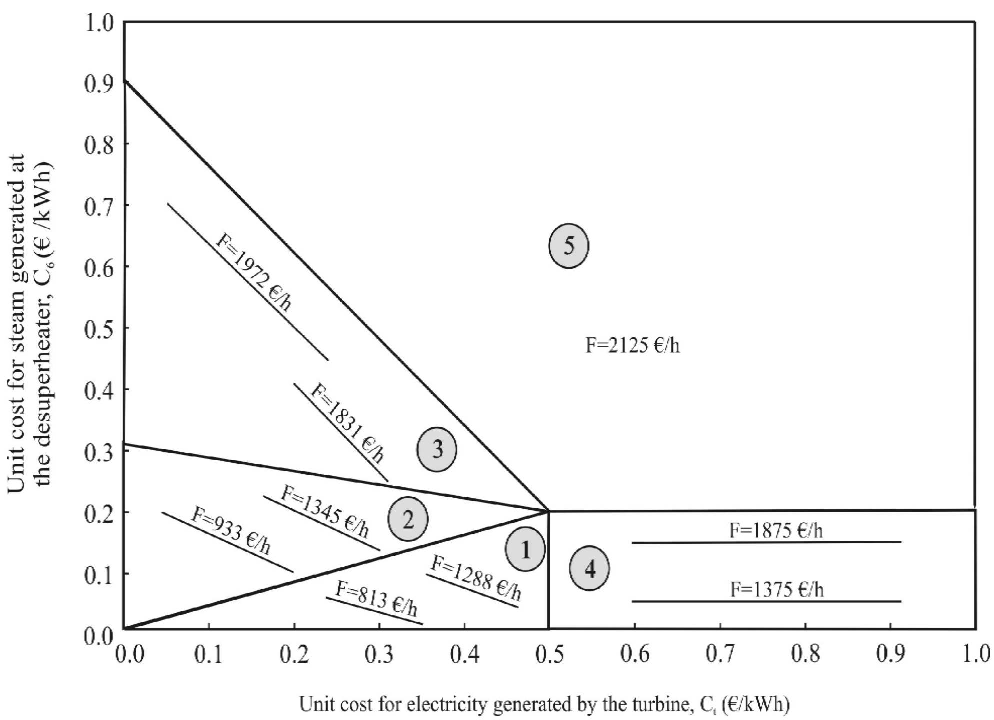

Figure 2. Cost-region chart for operation of the energy system: a particular region designates a use of a mix of steam shown in Fig. 3 and a mix of electricity shown in Fig. 4

Region 5: In region 5 of Fig. 2, the steam and electricity were purchased outside the factory (see Fig. 3 and 4). This was because the unit cost for the steam purchased from the outside supplier was lower than that of the steam that might be generated in the factory. The unit cost for electricity purchased from the line grid was lower than that of the electricity that might be generated in the power plant. As no energy was produced in the power plant, the boiler, the desuperheater, the turbine and the heat pump did not operate. This meaning that the operating costs in this entire region were 2125 €/h.

4. CONCLUSIONS

The LP method can be used as an operations-planning tool for existing steam-condensing systems to minimize operating costs. This is especially true when one or more power demands can be satisfied by either internal or external power sources. In this paper we present the LP optimization model for decreasing operating costs of the energy system that consisted of the boiler, the desuperheater, the turbine with steam extraction, and the heat pump. The cost-region chart which is derived for analyzed energy system depended on the unit prices of the different types of energy. The minimum

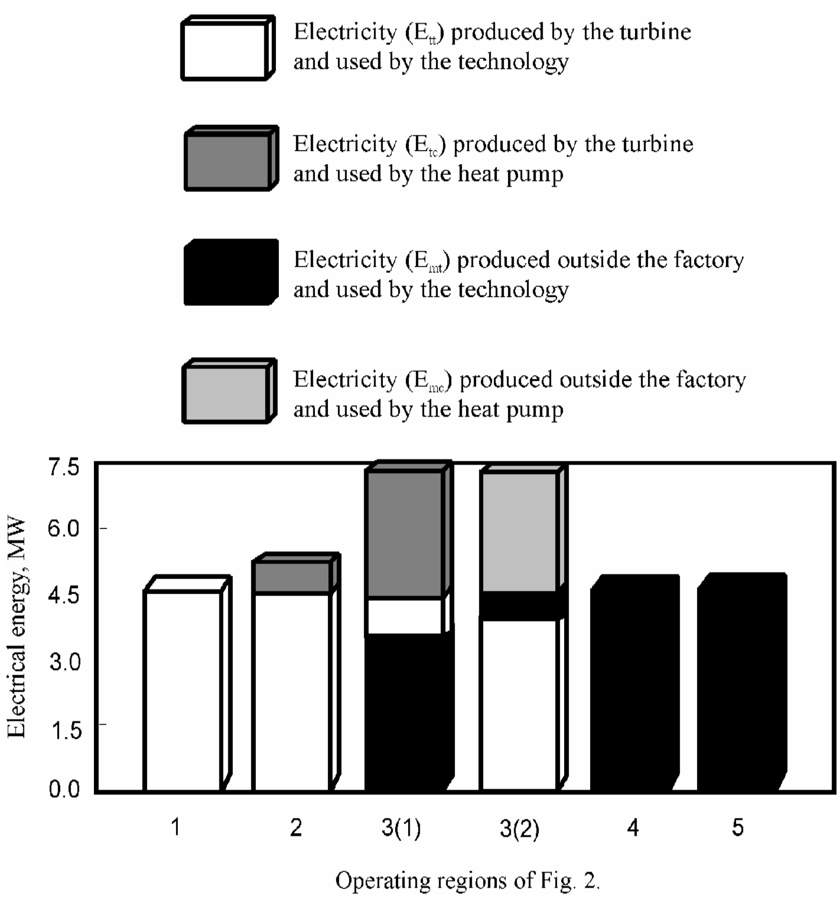

Figure 3. Different mixes of heat of the steam used by the technology in different operating regions of Fig. 2

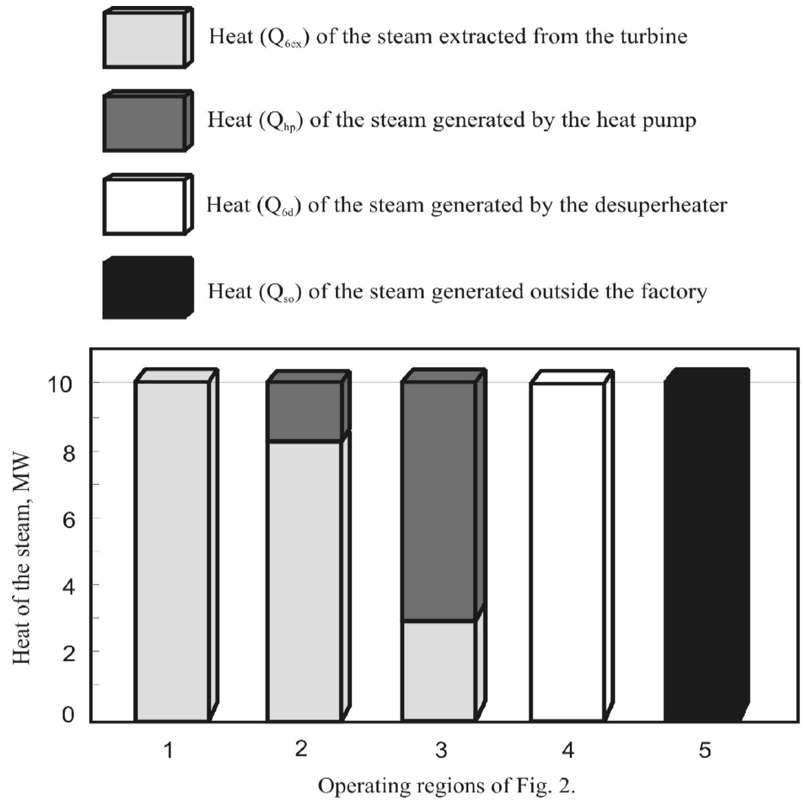

Figure 4. Different electricity mixes used by the technology and the heat pump in different operating regions of Fig. 2.

operating costs for energy use was recorded in region 1 of Fig. 2 where the technology energy consumption was completely satisfied by the power plant production. When the technology consumed outside source of heat or electricity, the maximum operating cost for energy use was recorded in region 4 of Fig. 3, i.e. 2125 €/h, and the minimum operating cost for energy use, in that case, was recorded in region 3 of Fig. 2, i.e. 867 €/h. Thus, when the system in region 3 can be operated on refuse fuel, a saving in operating cost of 1258 €/h, or 60%, may be realized, as compared to operation in region 5 of Fig. 3.

ПРИМЕНА ЛИНЕАРНОГ ПРОГРАМИРАЊА У УПРАВЉАЊУ ЕНЕРГИЈОМ

Снежана Драгићевић*[a] и Милорад Бојић[b]

[a] *Универзитет у Крагујевцу, Технички факултет у Чачку, Србија*
[b] *Универзитет у Крагујевцу, Машински Факултет у Крагујевцу, Србија*

Извод

Како се трошкови енергије и опреме повећавају, ефикасни енергетски системи постају све важнији у укупној економији процесних постројења. Овај рад представља метод моделовања и оптимизације индустријског система за кондензацију паре употребом техника линеарног програмирањљ (ЛП). Програм линеарног програмирања је употребљен да би се минимизирали укупни трошкови искоришћене енергије, као део укупних трошкова система кондензације паре. ЛП техника омогућује одређивање оптималних вредности за карактеристичне промењиве у дизајну процеса, како би се постигли минимални трошкови.

Кључне речи: Линеарно програмирањ, Енергетски менаџмент, Минимални оперативни трошкови.

References

Banks, J., Spoerer, J.P., & Collins, R.L. (1986). IBM PC Applications for the Industrial Engineer and Manager. Reston Book, New Jersey: Published by Prentice-Hall, Englewood Cliffs.

Bojić, M., & Dragićević, S. (2002) MILP optimization energy supply by using a boiler, a condensing turbine and a heat pump. Energy Conversion and Management 43: 591-608.

Bojić, M., & Stojanović, B. (1998) MILP optimization of a CHP energy system. Energy Conversion and Management, 39(7): 637-642.

Bojić, M., Stojanović, B., & Mourdoukoutas, P. (1998). MILP optimization of energy systems with a condensing turbine. Energy 23(3): 231-238.

Clark, J.K., & Helmick, J.K. (1982). How to Optimize the Design of Steam Systems, Process Energy Conversation, From R. Greene. New York: Mc Graw Hill.

Dragićević, S. (1998), Optimization of steam-condensing systems in industry, Master Thesis. Faculty of Mechanical engineering, University of Kragujevac, Serbia.

Grossman, I., & Santibanez, J. (1980) Applications of mixed-integer linear programming in process synthesis. Computers and Chemical Engineering, 4: 205-214.

Guldmann, J., & Wang, F. (1999) Optimizing the natural gas supply mix of local distribution utilities. European Journal of Operational Research, 112: 598-612.

Gustafsson, S.I., & Bojić, M. (1997) Optimal heating-system retrofits in residential buildings. Energy-The International Journal, 22(9): 867-874.

Spakovsky, M.R., Curti, V., & Batato, M. (1995) The performance optimization of a gas turbine cogeneration/heat pump facility with thermal storage. Journal of Engineering for Gas Turbines and Power, ASME transactions 117(3): 2-9.

Sundberg, J., & Wene, C.O. (1994) Integrated Modelling of Material Flows and Energy systems (MIMES). International Journal of Energy Research, 18: 359-381.

Vanderplaats, G.N. (1984). Numerical optimization techniques for engineering design with application. New York: McGraw-Hill.

Yokohama, R., & Ito, K. (2000) A novel decomposition method for MILP and its application to optimal operation of a thermal storage system. Energy Conversion and Management 41: 1781-95.