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SHORT-TERM SELLING OF A STOCK: A MODEL

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Abstract

This article presents a discrete period model for decision-making related to selling of the units of a stock. An exact analysis of the model is given and a method for determining optimal decisions is described. Application of the model is illustrated with some relevant numerical examples.

Keywords: Stock, Selling, Optimization.

1. INTRODUCTION

The decision problems related to purchasing and selling units of a stock (equity shares) are of high practical importance. Such problems include a number of factors and there can be many varying contexts. These problems are called as portfolio analysis & optimization, portfolio management, etc., in the relevant literature. Valuable works have been done by Markowitz (Markowitz, 1952), Sharpe (Sharpe, 1963), Perold (Perold 1984), Wilkie (Wilkie, 1986), among others, in this area. Further discussions on portfolio management are available, in (Elton et al. 2003), (Reilly & Brown, 2003), and in the texts by other authors.

As some recent expositions that are relevant to our subsequent discussion we may mention (Dokuchaev, 2008) and (Bayrakter & Young, 2010). In the first, the author considers a continuous time incomplete market model such that the risk-free rate, the appreciation rates and the volatility of the stocks are random. The distributions are unknown, but are observable currently. The optimal investment

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problem is stated in a "maximin" setting which leads to maximization of the minimum expected gain over all distributions of parameters of the related distributions of the random variables. In the second, the authors derive an optimal investment strategy to minimize the expected time that an individual's wealth stays below zero- called as the "occupation time." The individual consumes at a constant rate and invests in one risk-free and one risky asset, with the risky asset's price following a geometric Brownian motion. They also consider an extension of the model by penalizing the occupation time for the degree to which wealth is negative.

We propose in this article a model for optimal decision-making for selling some units of one stock. It is a simple model, which, nonetheless, can have some applications. The model, as a matter of fact, has some similarity with a commonly known riddle that may be described as in the following. In it, one gets offers of different values. The values, integers, can be between 1 and 1000 and no two offers have same value. If one accepts an offer, the game ends. Otherwise, next offer is placed. This continues for a fixed number of times. The gain is the value of the offer accepted. One has to adopt a strategy that maximizes his gain (in some sense). The model in this article differs from this in assuming a more general distribution of the values or prices, in assuming a trend component, etc. It allows an exact analysis. It differs from the two models stated earlier in the way that it is a discrete period model. The units must be sold within a given period. To the best of our knowledge, such a model, which can have various applications, has not been discussed in the literature.

We give the model and its analysis in the next section. Application of the model is illustrated with some numerical examples in the next. We conclude identifying some possible extension for the model and some other points.

2. MODEL & ANALYSIS

We first give the notation, used in this article.

2.1. Notation

n: Number of periods in the planning horizon;

$T(i)$: Trend component of the selling price of one unit of the stock in period $i = 1, 2, ..., n$;

ε_i: Random variable denoting uncertain variations in the selling price in period i;

$P(i)$: Selling price per unit in the i-th period;

x_i: Decision variable for period i – if the selling price per unit is observed to be higher than x_i the units are sold in the i-th period; otherwise, selling of the units is postponed;

$K(i)$: Maximum expected (average) gain, if the planning horizon would start from period i (extending up to period n);

$f(.)$: Probability density function (*pdf*) for the random variables ε_i;

$F(.)$: Distribution function (*df*) for the random variables;

$\overline{F}(.)$ Complement of the *df* $(\overline{F}(y) = 1 - F(y))$;

μ: Average of the random variables ε_i ($\mu = E[\varepsilon_i]$);

X: Random variable denoting the gain, given a decision.

2.2. Assumptions

We describe the assumptions made in the model, in the following:

(a) There are some units of a stock at the start. All of the units will be sold, waiting, at the maximum, for the next n periods;

(b) Selling price per unit in the i-th period is known as, $P(i) = T(i) + \varepsilon_i$; $P(i) \geq 0$;

(c) All the units can be sold;

(d) The random variables ε_i, $i = 1, 2, ..., n$, are mutually independent, identically distributed random variables with *pdf* $f(y)$, *df* $F(y)$ and average μ ($0 \leq \mu < \infty$);

(e) The *pdf* is given as, $f(y) > 0$, $a < y < b$ ($a \geq - min\{T(i)\}$); $f(y) = 0$, elsewhere;

(f) All of the units are to be sold on period i, if the selling price is observed to be higher than x_i, for the periods $i = 1, 2, ..., n-1$; if the units are not sold on or before the $(n-1)$-th period, these must be sold on the n-th period;

(g) The objective is to maximize the average gain, when the planning horizon starts from period 1, i.e., to determine $K(1)$ and the corresponding x_i, $i = 1, 2, ..., n-1$.

The model would be more appropriate in the short term, since the trends, the pdf of the prices are assumed to be known, rather than estimated. That is, estimation error is supposed to be negligible. It may be noted that, there is no justification in changing some decision variable values observing prices in the preceding periods, as these are mutually independent.

2.3. Analysis

We may write, using the independence of the random variables, the maximum average gains as,

$$K(i-1) = \max_{x_{i-1}}[\Pr\{\varepsilon_{i-1} > x_{i-1} - T(i-1)\}$$
$$[T(i-1) + \mathrm{E}[\varepsilon_{i-1} \mid \varepsilon_{i-1} > x_{i-1} - T(i-1)]]$$
$$+ \Pr\{\varepsilon_{i-1} \leq x_{i-1} - T(i-1)\}K(i)],$$
$$i = 2, 3, ..., (n-1); \tag{1}$$

$$K(n-1) = \max_{x_{n-1}}[\Pr\{\varepsilon_{n-1} > x_{n-1} - T(n-1)\}$$
$$[T(n-1) + \mathrm{E}[\varepsilon_{n-1} \mid \varepsilon_{n-1} > x_{n-1} - T(n-1)]]$$
$$+ \Pr\{\varepsilon_{n-1} \leq x_{n-1} - T(n-1)\}(T(n) + \mu)] \tag{2}$$

To determine $K(n-1)$, we need to maximize, with respect to x_{n-1},

$$\overline{F}(x_{n-1} - T(n-1))[T(n-1) + \frac{1}{\overline{F}(x_{n-1} - T(n-1))}$$
$$\int_{x_{n-1}-T(n-1)}^{b} yf(y)dy] + F(x_{n-1} - T(n-1))(T(n) + \mu)$$
$$= \overline{F}(x_{n-1} - T(n-1))T(n-1) + \int_{x_{n-1}-T(n-1)}^{b} yf(y)dy$$
$$+ F(x_{n-1} - T(n-1))(T(n) + \mu) \tag{3}$$
$$= g(x_{n-1}).$$

Setting the derivative dg/dx_{n-1} equal to zero gives,

$$f(x_{n-1} - T(n-1))(T(n) + \mu - x_{n-1}) = 0 \tag{4}$$

$$\text{i.e.,} \quad x_{n-1} = T(n) + \mu. \tag{5}$$

Thus, if a solution exists for $dg/dx_{n-1} = 0$, $T(n-1) + a < x_{n-1} < T(n-1) + b$, it is unique. Verify the second derivative $\mathrm{d}^2 g/\mathrm{d}\, x_{n-1}^2$ (assuming that $\mathrm{d}f/dx = f^{/}(.)$ exists at the point). This is given as,

$$-f^{/}(x_{n-1} - T(n-1))T(n-1) - f(x_{n-1} - T(n-1))$$
$$-(x_{n-1} - T(n-1))f^{/}(x_{n-1} - T(n-1))$$
$$+ f^{/}(x_{n-1} - T(n-1))(T(n) + \mu) \tag{6}$$

$$= -f(x_{n-1} - T(n-1)) + f^{/}(x_{n-1} - T(n-1))$$

$$(T(n) + \mu - x_{n-1}) < 0. \quad (7)$$

In such a case, the solution is the unique maximum point. If the derivative is zero at a single point, but optimality cannot be checked with second derivative, then the solution can be compared with the solutions as $x_{n-1} = T(n-1) + a$, $x_{n-1} = T(n-1) + b$.

If the derivative is not zero at any point such that, $a < (x_{n-1} - T(n-1)) < b$, and $a > (x_{n-1} - T(n-1))$ from (5), then optimally, $(x_{n-1} - T(n-1)) = a$.

The case $(x_{n-1} - T(n-1)) > b$, from (4), would require that, $(x_{n-1} - T(n-1)) = b$.

If $(x_{n-1} - T(n-1)) = a$, that would mean the units must be sold in the $(n-1)$-th period; if $(x_{n-1} - T(n-1)) = b$, the units must not be sold in this period.

We may determine $K(i)$, $i = (n-2), (n-3), ..., 1$ in the same way, since, we only have to consider $K(i+1)$ in the place of $(T(n) + \mu)$. That is, we need to maximize,

$$\overline{F}(x_i - T(i))T(i) + \int_{x_i - T(i)}^{b} yf(y)dy$$

$$+ F(x_i - T(i))K(i+1) \quad (8)$$

And we would obtain,

$$x_i = K(i+1) \quad (9)$$

We may also state the following proposition for the model.

Proposition 2.1: For an optimal solution,

i. if there is a non-increasing trend as,

$T(1) \geq T(2) \geq ... \geq T(n-1) \geq T(n)$, then,

$x_1 \geq x_2 \geq ... \geq x_{n-1}$;

ii. if there is a non-decreasing trend as,

$T(1) \leq T(2) \leq ... \leq T(n-1)$, then,

$x_1 - T(1) \geq x_2 - T(2) \geq ... \geq x_{n-1} - T(n-1)$.

Proof: *i*. In an optimal solution it must hold that,

$$K(1) \geq K(2) \geq ... \geq K(n-1) \geq K(n) = T(n) + \mu,$$

since there exists a solution $x_i - T(i) = b$ ($i = 1, 2, ..., (n-1)$) (i.e., do not sell in the i-th period), such that, $K(i) = K(i+1)$. Consider two adjacent solutions x_i and x_{i+1} ($i = 1, 2, ..., (n-2)$). If both the solutions satisfy (9), then we have $x_i \geq x_{i+1}$. If x_i does not satisfy (9), then, from (4), $x_i = T(i) + a$. The same holds for x_{i+1}. Thus, if x_{i+1} does not satisfy (9), $x_i \geq x_{i+1}$. If x_{i+1} satisfies (9) and x_i does not, $x_{i+1} = K(i+2)$, $x_i = T(i) + a$. Since, x_i does not satisfy (9),

$K(i+1) < T(i) + a$. As $K(i+2) \leq K(i+1)$, $x_i > x_{i+1}$

ii. Done in the same way as in *i*, using (8).

According to the proposition, if there is a non-increasing trend, optimal selling price should be highest for the first period, then should decrease. For non-decreasing trend, this holds for $x_i - T(i)$ values. Such observations, however, may not hold for any general trend. But, maximum expected gain increases with the length of planning horizon, independent of the trend.

We may also note that, the analysis remains, in effect, the same in the following cases.

(a) It is allowed that, all of the units need not to be sold simultaneously. Clearly, if expected gain per unit is maximized for a solution, all the units can be sold as given by it. There is no need to sell amounts, in parts, in different periods.

(b) Instead of maximizing expected gain, we may also consider maximizing the expectation of a function as, $h(X)$ of the gain. This would allow us, for instance, to consider expected gain minus the variance of the gain, multiplied with a positive constant. The solution, however, may change from a solution as obtained with the present model.

(c) Prices are non-identically distributed, but independent.

3. NUMERICAL EXAMPLES & OBSERVATIONS

We take the following examples, considering uniform and normal distributions, to illustrate the application of the model. The experiment has been done in *MS Excel.* For the case of normal distribution, the integral for conditional expectation (3) has been calculated numerically with a computer routine written in *Visual Basic.* For each case of the examples, we also calculate the standard deviation of the gain for the optimal solution, and, expected gain and standard deviation of gain for a comparative solution. The comparative solution is obtained by setting $x_i = T(i) + \mu$ i.e., at average prices. In the tables, values have been rounded to the second digit after decimal point.

(a) Suppose that, there is no trend and ε_i are uniformly distributed in (80.0, 130.0). The units are to be sold within next 7 periods, e.g., days. The distribution is with mean 105.0 and standard deviation 14.43. The results are shown in Table 1. As can be seen, the selling price on which the units are to be sold is highest on the first day, and it gradually decreases. Maximum expected gain increases as there is more time in the planning horizon. Standard deviation of gain, at maximum expected gain, is not high relative to that of the comparative solution.

(b) In the same set up, consider that there is a positive linear trend given as, $T(i) = 5i$, that is, price increases, in part, certainly by 5 units every day. Optimal selling prices become higher, compared to the earlier case. Standard deviation of the solution compares favorably with that of the comparative solution. The results are shown in Table 2.

(c) Consider a negative linear trend as, $T(i) = -5i$. In this instance, optimal selling prices are reduced than that in both of the earlier cases. Standard deviation is sometimes lower than that of the comparative solution. The results are reported in Table 3.

(d) We consider that, random variables ε_i follow a (truncated) normal distribution with mean 105.0 and standard deviation 10.0 and there is a positive trend $T(i) = 5i$. The results are given in Table 4. Compared to the same conditions with uniform distribution (Example (b), Table 2), maximum expected gain is slightly reduced. This may be explained with the fact that higher values of selling price have less likelihood in this case. In this case also, standard deviation of gain, for the optimal solution is not high, whereas expectation of gain is considerably more than that in the comparative solution.

4. CONCLUDING REMARKS

We have presented a simple, intuitively appealing model for optimal decision-making related to selling some units of a stock. Selling price varies randomly, with a

Table 1. Optimal Selling Prices without Trend (Uniform Distribution)

Obs. No.	**From Period** (i)	**Trend** $T(i)$	**Comparative Solution** (Selling Price)	**Expected Gain** (Standard Dev.)	**Maximizing Selling Price** (x_i)	**Expected Gain** ($K(i)$) (Standard Dev.)
1	7	0	-	105.00 (14.43)	-	105.00 (14.43)
2	6	0	105.00	111.25 (13.01)	105.00	111.25 (13.01)
3	5	0	105.00	114.38 (10.97)	111.25	114.77 (11.72)
4	4	0	105.00	115.94 (9.42)	114.77	117.09 (10.66)
5	3	0	105.00	116.72 (8.43)	117.09	118.75 (9.79)
6	2	0	105.00	117.11 (7.85)	118.75	120.02 (9.07)
7	1	0	105.00	117.30 (7.55)	120.02	121.02 (8.45)

Table 2. Optimal Selling Prices with a Positive Trend (Uniform Distribution)

Obs. No.	**Period** (i)	**Trend** $T(i)$	**Comparative Solution** (Selling Price)	**Expected Gain** (Standard Dev.)	**Maximizing Selling Price** (x_i)	**Expected Gain** ($K(i)$) (Standard Dev.)
1	7	35	-	140.00 (14.43)	-	140.00 (14.43)
2	6	30	135.00	143.75 (12.01)	140.00	144.00 (12.74)
3	5	25	130.00	143.13 (9.93)	144.00	145.21 (11.58)
4	4	20	125.00	140.31 (9.12)	145.21	145.44 (11.04)
5	3	15	120.00	136.41 (9.11)	145.00	145.44 (11.04)
6	2	10	115.00	131.95 (9.35)	140.00	145.44 (11.04)
7	1	5	110.00	127.23 (9.59)	135.00	145.44 (11.04)

possible trend, over the periods. There is a maximum number of periods within which the units must be sold.

A method to obtain optimized selling prices, to maximize the expected gain, is given. Application of the model has been illustrated with a few numerical examples. The numerical experiment indicates that, solutions yielded with the method would be practically suitable, as average gain is maximized without having too high standard deviation.

Many extensions and variations are possible for the present model. Some of these

Table 3. Optimal Selling Prices with a Negative Trend (Uniform Distribution)

Obs. No.	**Period** (i)	**Trend** $T(i)$	**Comparative Solution** (Selling Price)	**Expected Gain** (Standard Dev.)	**Maximizing Selling Price** (x_i)	**Expected Gain** ($K(i)$) (Standard Dev.)
1	7	-35	-	70.00 (14.43)	-	70.00 (14.43)
2	6	-30	75.00	78.75 (14.38)	70.00	79.00 (13.50)
3	5	-25	80.00	85.63 (13.29)	79.00	85.76 (12.61)
4	4	-20	85.00	91.56 (12.23)	85.76	91.64 (11.93)
5	3	-15	90.00	97.03 (11.44)	91.64	97.09 (11.45)
6	2	-10	95.00	102.27 (10.90)	97.09	102.34 (11.12)
7	1	-5	100.00	107.38 (10.57)	102.34	107.48 (10.90)

Table 4. Optimal Selling Price with a Positive Trend (Normal Distribution)

Obs. No.	**From Period** (i)	**Trend** $T(i)$	**Comparative Solution** (Selling Price)	**Expected Gain** (Standard Dev.)	**Maximizing Selling Price** (x_i)	**Expected Gain** ($K(i)$) (Standard Dev.)
1	7	35	-	140.00 (10.00)	-	140.00 (10.00)
2	6	30	135	141.51 (8.25)	140.00	142.00 (9.17)
3	5	25	130	139.77 (7.27)	142.00	142.56 (8.85)
4	4	20	125	136.39 (7.32)	142.56	142.73 (8.88)
5	3	15	120	132.21 (7.75)	142.73	142.77 (8.63)
6	2	10	115	127.61 (8.17)	142.77	142.78 (8.62)
7	1	5	110	122.82 (8.49)	142.78	142.78 (8.61)

are analyzed essentially in the same way as has been done for the present model. Some other extensions may be as:

(a) To consider that, the prices are correlated;

(b) Considering a continuous version of the problem. The prices may be assumed to follow a Brownian motion/ geometric Brownian motion process with a drift, or as may be appropriate;

(c) Considering that, prices are not known but are estimates. The estimates may also be refined subsequently with more information, if the units are not already sold.

Such extensions will further enhance the applicability of the model, and future research work of this nature will be highly valuable.

КРАТКОРОЧНА ПРОДАЈА АКЦИЈА: МОДЕЛ

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Извод

Овај рад представља модел доношења одлука, у оквиру дискретног временског периода, везано за продају јединице акција. Дата је егзактна анализа модела и описан метод за одређивање оптималне одлуке. Примена модела је илустрована на релевантним нумеричким примерима.

Кључне речи: Акција, продаја, оптимизација.

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